Review of Multivariate Calculus

In what follows, u, w represent scalar functions and $\mathbf{v} = (v_1, v_2, v_3)$ represents a vector function. n represents the unit outward normal vector, to the boundary ∂R.

$$gradient\ u \equiv \nabla u = (\tfrac{\partial}{\partial x}, \tfrac{\partial}{\partial y}, \tfrac{\partial}{\partial z})u = (\tfrac{\partial u}{\partial x}, \tfrac{\partial u}{\partial y}, \tfrac{\partial u}{\partial z})$$

$$divergence\ \mathbf{v} \equiv \nabla \bullet \mathbf{v} = (\tfrac{\partial}{\partial x}, \tfrac{\partial}{\partial y}, \tfrac{\partial}{\partial z}) \bullet \mathbf{v} = \tfrac{\partial v_1}{\partial x} + \tfrac{\partial v_2}{\partial y} + \tfrac{\partial v_3}{\partial z}$$

$$\nabla \bullet \nabla u = \tfrac{\partial^2 u}{\partial x^2} + \tfrac{\partial^2 u}{\partial y^2} + \tfrac{\partial^2 u}{\partial z^2} \equiv \nabla^2 u$$

Divergence theorem:

$$\iiint\limits_R \nabla \bullet \mathbf{v} = \iint\limits_{\partial R} \mathbf{v} \bullet n$$

Product rules (second is just first with $\mathbf{v} = \nabla w$)

$$\nabla \bullet (u\mathbf{v}) = u\nabla \bullet \mathbf{v} + \nabla u \bullet \mathbf{v}$$

$$\nabla \bullet (u\nabla w) = u\nabla^2 w + \nabla u \bullet \nabla w$$

Integration by parts (follow from product rules and divergence theorem):

$$\iiint\limits_R u\nabla \bullet \mathbf{v} = \iint\limits_{\partial R} u\mathbf{v} \bullet n - \iiint\limits_R \nabla u \bullet \mathbf{v}$$

$$\iiint\limits_R u\nabla^2 w = \iint\limits_{\partial R} u\nabla w \bullet n - \iiint\limits_R \nabla u \bullet \nabla w = \iint\limits_{\partial R} u\tfrac{\partial w}{\partial n} - \iiint\limits_R \nabla u \bullet \nabla w$$