On "compensating" entropy decreases

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Abstract: The "compensation" argument, widely used to dismiss the claim that evolution violates the more general statements of the second law of thermodynamics, is based on the idea that there is a single quantity called "entropy" which measures disorder of all types. This article shows that there is no such total entropy, and that the compensation argument is not a valid way to dismiss the claim that evolution violates the second law. Note that the article does not argue that evolution violates the second law, only that the compensation argument is logically invalid. © *2017 Physics Essays Publication*. [http://dx.doi.org/10.4006/0836-1398-30.1.70]

Résumé: L'argument de la "compensation," qui est souvent utilisé pour rejeter l'affirmation que l'évolution enfreint les déclarations plus générales de la deuxième loi de la thermodynamique, se base sur l'idée qu'il existe une quantité unique appelée "entropie," qui mesure le désordre de tout genre. Cet article démontre que l'entropie totale n'existe pas et que l'argument de la compensation n'est pas une manière valable pour rejeter l'affirmation que l'évolution enfreint la deuxième loi. Veuillez noter que le but de l'article n'est pas de déclarer que l'évolution enfreint la deuxième loi, mais plutôt que l'argument de la compensation n'est pas valable selon la logique.

Key words: Entropy; Second Law of Thermodynamics; Evolution.

The idea that "entropy" is a single quantity which measures disorder of all types is widely believed.

Of course, you can define a quantity, which I will call "thermal" entropy, which measures randomness (uniformness) in the temperature distribution, and show that in an isolated system this thermal entropy can only increase, as heat diffuses and the temperature distribution becomes more and more uniform. And you can similarly define an "X-entropy" which measures randomness in the distribution of any other diffusing component X and show, using the same equations, that in an isolated system this X-entropy also can only increase [Eq. (A8) in the Appendix], as the component X diffuses and the distribution of X becomes more and more uniform. But the idea that there is a total entropy which measures randomness of all types is a myth, and is the source of much confusion.

Carnap,[1] in *Two Essays on Entropy*, writes "There are many thermodynamic entropies, corresponding to different degrees of experimental discrimination and different choices of parameters. For example, there will be an increase in entropy by mixing samples of ^{16}O and ^{18}O only if isotopes are experimentally distinguished."

Imagine an isolated box in which, initially, the left half is pure ^{16}O, the right half is pure ^{18}O, at the same pressure and temperature. As time passes, the isotopes mix, and eventually the isotopes are randomly distributed throughout the box. Has "entropy" increased? If the only entropy you recognize is thermal entropy, there is no entropy change (see Ref. 2, Fig. 1-IV, pp. 353–354). Or if you think of entropy as disorder (randomness) in the distribution of oxygen, and do not notice that there are different isotopes present, this entropy was also already maximal, and so there was no further increase with time. But if you think of entropy as disorder in the distribution of ^{18}O, then entropy has certainly increased. Does it make any sense to combine these types of entropy and say the second law of thermodynamics only requires that total entropy must increase? (How many other types of entropy need to be added to get a total entropy?) By that logic, you could say that oxygen in general could become less uniformly distributed, with a resultant decrease in oxygen entropy, as long as this decrease is compensated by an even greater increase in ^{18}O entropy. For that matter, you might say that oxygen and thermal entropies could both decrease as long as the ^{18}O entropy increase is greater. But of course, the second law would not really allow either type of entropy to decrease, because it is extremely improbable that either would decrease in this isolated system. So, it makes no sense to combine different types of entropy into a single quantity that measures all types of disorder, even when entropy is quantifiable, such as in this example.

Nevertheless, the existence of this elusive quantity is widely assumed. Urone, in *College Physics*[3] writes:

> Some people misuse the second law of thermodynamics, stated in terms of entropy, to say that the existence and evolution of life violate the law and thus require divine intervention. It is true that the evolution of life from inert matter to its present forms represents a large decrease in entropy for living systems. But it is always possible for the entropy of one part of the universe to decrease,

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provided the total change in entropy of the universe increases.

The authors of *Order and Chaos*[4] similarly write:

In a certain sense the development of civilization may appear contradictory to the second law. Even though society can effect local reductions in entropy, the general and universal trend of entropy increase easily swamps the anomalous but important efforts of civilized man. Each localized, man-made or machine-made entropy decrease is accompanied by a greater increase in entropy of the surroundings, thereby maintaining the required increase in total entropy.

In a 2008 *American Journal of Physics* article,[5] Styer estimates the rate of decrease in entropy due to evolution by assuming that "each individual organism is 1000 times more improbable than the corresponding individual was 100 years ago," which he calls a "very generous" assumption. Then he uses the Boltzmann formula[b)] to calculate that a 1000-fold decrease in probability corresponds to an entropy decrease of $k_B \log(1000)$, multiplies this by a generous overestimate for the number of organisms on Earth, and divides by the number of seconds in a century to estimate that the rate of decrease in entropy due to evolution is less than about 302 J/K/s, a rate of decrease which is orders of magnitude less than the "Earth's entropy throughput." Thus, he concludes, there is no conflict with the second law because "presumably the entropy of the Earth's biosphere is indeed decreasing by a tiny amount due to evolution and the entropy of the cosmic microwave background is increasing by an even greater amount to compensate for this decrease." Since about five million centuries have passed since the beginning of the Cambrian era, if organisms are, on average, a thousand times more improbable every century, that would mean that today's organisms are about $10^{15000000}$ times more improbable than those early organisms. But, according to this compensation argument, there is no conflict with the second law because the Earth is an open system, and something is happening outside the Earth (thermal entropy is increasing) which, if reversed, would be even more improbable. In a later *American Journal of Physics* article, Bunn[6] concludes that Styer's assumption that organisms are 1000 times more improbable every century was not really generous, that a factor of 10^{25} each century is more reasonable, but shows that, still, "$(dS/dt)_{\rm sun} + (dS/dt)_{\rm life} \geq 0$" so "the second law of thermodynamics is safe."

To see what is wrong with this argument, suppose a tornado turns a town into rubble, then a second tornado turns this rubble back into houses and cars. Let us generously assume that a house is $10^{1000000000}$ times "more improbable" than a pile of rubble (i.e., assume there are $10^{1000000000}$ "rubble" microstates for every "house" microstate) and that there are 10000 houses in the town, and that it takes the second tornado 5 min to reconstruct the town. If we use the Boltzmann formula to calculate the rate of decrease in entropy caused by the second tornado, we get a rate of about $k_B \log([10^{1000000000}]^{10000})/(5*60) = 10000 k_B \log(10^{1000000000})/(300) \approx 10^{-12}$ J/K/s, about 14 orders of magnitude less even than the rate for evolution estimated by Styer. The town is an open system, since tornados derive their energy from the sun, and the total entropy of the universe still increases, so the second tornado does not pose any conflict with the second law, by Styer's logic.[c)]

The "compensation" idea, used by Urone and Styer and Bunn, and every physics textbook which discusses the second law and evolution, is not reasonable even for quantifiable measures of entropy, and even for isolated systems, as the oxygen mixing example illustrates.

In any case, it is easy to show (see the Appendix) that the equations of entropy change do not only say that thermal entropy cannot decrease in an isolated system, they also say that, in an open system, the thermal entropy cannot decrease faster than it is exported through the boundary, and the X-entropy cannot decrease faster than it is exported through the boundary. Stated another way, the "X-order" (defined as the negative of X-entropy) in an open system cannot increase faster than it is imported through the boundary. This I showed first in a letter to the editor of *The Mathematical Intelligencer*,[7] then in an appendix to my John Wiley book *The Numerical Solution of Ordinary and Partial Differential Equations, 2nd edition*,[8] and then in a 2011 submission to *Applied Mathematics Letters* (AML).

Thus, I argued, the equations of entropy change do not support the illogical compensation argument, instead they illustrate the tautology that "if an increase in order is extremely improbable when a system is isolated, it is still extremely improbable when the system is open, unless something is entering which makes it *not* extremely improbable." Thus, to argue that evolution does not violate the second law, you cannot simply dismiss the problem by saying, the Earth is an open system so any decreases in entropy here are easily compensated by increases elsewhere, you have to argue that thanks to the influx of solar energy, it is not really impossibly improbable that the four fundamental, unintelligent forces of physics alone could rearrange the fundamental particles of physics into computers, science libraries, airplanes, and iPhones. Common sense tells us that the fact that order can increase in an open system does not mean that tornados can turn rubble into houses and cars, or that computers can appear on a barren planet as long as the planet receives solar energy. Something must be entering the open system which makes the appearance of computers not extremely improbable, for example, computers.

The *AML* submission was reviewed and accepted, and then withdrawn by the journal about a week before it was to appear, with the explanation from the editor that "our editors simply found that it does not consist of the kind of content

[b)]The Boltzmann formula is used to calculate the relative probabilities of two ideal gas states with a given thermal entropy difference, but this does not mean it can be used to convert *any* probability change into units of thermal entropy. This is like finding a function p(x) which gives the probability of living x years or longer, and saying, now we know how to convert the probability of *anything* into its equivalent in years!

[c)]Whether my wild guesses at the probabilities involved are generous or not, the second law is certainly safe as long as the second tornado destroys two still intact houses for every one it reconstructs, according to this logic.

we are interested in publishing." The typeset version can be viewed at www.math.utep.edu/Faculty/sewell/AML_3497.pdf. The editor later published a formal apology in the journal[9] stating that the accepted article was withdrawn "not because of any errors or technical problems found by the reviewers or editors, but because the Editor-in-Chief subsequently concluded that the content was more philosophical than mathematical."

One can precisely define entropies which are useful in simple situations such as diffusion, but when we try to apply the second law in more general situations,"entropy" is nothing more or less than a scientific-sounding synonym for "disorder" and is no more precise than disorder, and it seems to be primarily useful to physics textbook writers as a way to avoid the issue of probability when discussing evolution and the second law. But the second law is always about probability, so what is still useful in more complicated scenarios is the fundamental principle behind all applications of the second law, which is that natural causes do not do *macroscopically* describable things which are extremely improbable from the *microscopic* point of view.[d)] And in an open system, you just have to take into account what is crossing the boundary in deciding what is extremely improbable and what is not. When thermal entropy decreases in an open system, there is not anything macroscopically describable happening which is extremely improbable from the microscopic point of view, something is just entering the open system which makes the decrease not extremely improbable. This is the only principle that is useful when applying the second law to tornados or evolution or airplane crashes. It still makes sense to say, for example, that the second law—or at least the fundamental natural principle behind this law—predicts that natural causes (rust, fire, tornados, crashes) can turn airplanes into junk metal but not vice versa, because there are very few arrangements of atoms which could transport passengers through the air over long distances, and very many which could not. And perhaps you could say that there is a particular type of entropy, or "disorder," even if difficult to define or measure, which increases when planes crash. But note that this entropy has little or nothing to do with any X-entropy or thermal entropy, so trying to add the change in entropy due to a plane crash to changes in thermal entropy, to estimate a total entropy change, is a meaningless exercise. Why would this entropy even have units of thermal entropy? Even X-entropy does not, as shown in footnote e.

In 2012 *The Mathematical Intelligencer* published an article by Lloyd[10] which criticized my writings on this topic, primarily the *still unpublished* AML paper. Lloyd wrote "Although there is a local decrease in entropy associated with the appearance and evolution of life on Earth, this is very small in comparison with the very large entropy increase associated with the solar input to Earth. This qualitative idea has received quantitative backing from the calculations of Styer, and particularly as modified by Bunn, which show that the solar contribution is many orders of magnitude larger than any possible decrease associated with evolution." Lloyd wrote that I claim that my X-entropies always behave independently, that "this point is central to all the versions of his argument." Then he shows that under certain conditions they are not independent, and thus "the separation of total entropy into different entropies is invalid." In fact, I never claimed or believed that these X-entropies always behave independently, I made it clear in the *AML* paper that my conclusions "are only valid for our simple models, where it is assumed that only heat conduction or diffusion is going on; naturally, in more complex situations, the laws of probability do not make such simple predictions." But in this simple model the different entropies are indeed independent and thus nicely illustrate the common-sense conclusion that "if an increase in order is extremely improbable when a system is isolated, it is still extremely improbable when the system is open, unless something is entering which makes it *not* extremely improbable," which *is* the central point of my argument.

Of course, the reason my accepted *AML* paper was withdrawn was because it seemed to support intelligent design (ID) theory. In fact, I am a known ID supporter, and have even written a book on the topic.[11] I can certainly understand why many scientists feel that publishing anything supporting ID is not appropriate for a science journal, but the *AML* article did not *explicitly* promote intelligent design. Here were my conclusions in that paper: "Of course, one can still argue that the spectacular increase in order seen on Earth does not violate the second law because what has happened here is not really extremely improbable. And perhaps it only seems extremely improbable, but really is not, that, under the right conditions, the influx of stellar energy into a planet could cause atoms to rearrange themselves into nuclear power plants and spaceships and digital computers. But one would think that at least this would be considered an open question, and those who argue that it really is extremely improbable, and thus contrary to the basic principle underlying the second law of thermodynamics, would be given a measure of respect, and taken seriously by their colleagues, but we are not."

If Darwin was right, then evolution does not violate the second law because, thanks to natural selection of random mutations, and to the influx of stellar energy, it is not really impossibly improbable that advanced civilizations could spontaneously develop on barren, Earth-like planets. Getting rid of the compensation argument would not change that; what it might change is, maybe science journals and physics texts will no longer say, sure, evolution is astronomically improbable, but there is no conflict with the second law because the Earth is an open system, and things are happening elsewhere which, if reversed, would be even more improbable.

[d)]Extremely improbable events must be macroscopically (simply) describable to be forbidden; if we include extremely improbable events which can only be described by an atom-by-atom accounting, there are so many of these that some are sure to happen. (If we flip a billion fair coins, any particular outcome we get can be said to be extremely improbable, but we are only astonished if something extremely improbable and simply describable happens, such as "the last million coins are tails.") If we define an event to be simply describable when it can be described in m or fewer bits, there are at most 2^m simply describable events; then we can set the probability threshold for an event to be considered "extremely improbable" so low that we can be confident that *no* extremely improbable, simply describable events will ever occur.

APPENDIX: THE EQUATIONS OF ENTROPY CHANGE

Consider the diffusion (conduction) of heat in a region, R, with absolute temperature distribution $T(x, y, z, t)$. Conservation of energy requires that

$$\frac{\partial Q}{\partial t} = -\nabla \cdot J, \tag{A1}$$

where Q is the heat energy density $(\partial Q/\partial t = c\rho(\partial T/\partial t))$ and J is the heat flux vector. The second law requires that the heat flux be in a direction in which the temperature is decreasing, i.e.,

$$J \cdot \nabla T \leq 0. \tag{A2}$$

The above equation simply says that heat flows from hot to cold regions—because the laws of probability favor a more uniform temperature distribution.

"Thermal entropy" is a quantity that is used to measure randomness in the temperature distribution. The rate of change of thermal entropy, S, is given by the usual definition as

$$\frac{dS}{dt} = \iiint_R \frac{(\partial Q/\partial t)}{T} dV. \tag{A3}$$

Using Eqs. (A3) and (A1), after doing a (multidimensional) integration by parts, we get

$$\frac{dS}{dt} = \iiint_R \frac{-J \cdot \nabla T}{T^2} dV - \iint_{\partial R} \frac{J \cdot n}{T} dA, \tag{A4}$$

where n is the outward unit normal on the boundary ∂R. From the second law (A2), we see that the volume integral is nonnegative, and so

$$\frac{dS}{dt} \geq -\iint_{\partial R} \frac{J \cdot n}{T} dA. \tag{A5}$$

From Eq. (A5), it follows that $dS/dt \geq 0$ in an isolated system, where there is no heat flux through the boundary $(J \cdot n = 0)$. Hence, in an isolated system, the entropy can never decrease. Since thermal entropy measures randomness (disorder) in the temperature distribution, its opposite (negative) can be referred to as "thermal order," and we can say that the thermal order can never increase in an isolated system.

Since thermal entropy is quantifiable, the application of the second law to thermal entropy is commonly used as the model problem on which our thinking about the other, less quantifiable, applications is based. The fact that thermal entropy cannot decrease in an isolated system, but can decrease in a non-isolated system, was used to conclude that, in other applications, any entropy decrease in a non-isolated system is possible as long as it is compensated somehow by entropy increases outside this system, so that the total "entropy" (as though there were only one type) in the universe, or any other isolated system containing this system, still increases.

However, there is really nothing special about "thermal" entropy. Heat conduction is just diffusion of heat, and we can define an "X-entropy" S_X (and an X-order $= -S_X$), to measure the randomness in the distribution of any other substance X that diffuses: for example, X might be chromium diffusing in steel (again we assume nothing is going on but diffusion). If $C(x, y, z, t)$ represents the density (concentration) of X, we can define X-entropy [cf. Eq. (A3)] by[e)]

$$\frac{dS_X}{dt} = \iiint_R \frac{(\partial C/\partial t)}{C} dV \tag{A6}$$

and now conservation of X implies $\partial C/\partial t = -\nabla \cdot J$ [cf. Eq. (A1)], where J is now the flux of X, and integration by parts again yields [cf. Eq. (A4)]

$$\frac{dS_X}{dt} = \iiint_R \frac{-J \cdot \nabla C}{C^2} dV - \iint_{\partial R} \frac{J \cdot n}{C} dA. \tag{A7}$$

And since the X flux must be in a direction in which the X concentration is decreasing, $J \cdot \nabla C \leq 0$ [cf. Eq. (A2)], so

$$\frac{dS_X}{dt} \geq -\iint_{\partial R} \frac{J \cdot n}{C} dA, \tag{A8}$$

which now says that the X-entropy cannot decrease in an isolated system.

Furthermore, Eq. (A8) does not simply say that the X-entropy cannot decrease in an isolated system; it also says that in a nonisolated system the X-entropy cannot decrease faster than it is exported through the boundary [Eq. (A5) says the thermal entropy cannot decrease faster than it is exported], because the boundary integral there represents the rate at which X-entropy is exported across the boundary. To see this, notice that without the denominator C, the integral in Eq. (A6) represents the rate of change of total X in the system; with the denominator, it represents the rate of change of X-entropy. Without the denominator, C, the boundary integral in Eq. (A8) represents the rate at which X is exported through the boundary; with the denominator, therefore it must represent the rate at which X-entropy is exported. Although I am certainly not the first to recognize that the boundary integral has this interpretation (see Ref. 12, p. 202),[f)] this has been noticed by relatively few

[e)]Note that with this definition, specific (per unit volume) X-entropy is dimensionless, and the only way to get an X-entropy in units of thermal entropy is to multiply by an arbitrary constant with units of energy/temperature/volume. One might alternatively suggest that we could define a dimensionless thermal entropy with rate of change $\iiint (\partial T/\partial t)/T dV$, and if specific heat and density were constant everywhere that would work, but since they are not, this would not be a valid entropy because it would no longer be guaranteed to only increase in an isolated system. For example, if two blocks of equal volume are brought into contact, a hot block with low heat capacity and a cold block of very high heat capacity, the entropy of this system would decrease, using this definition of entropy. Thus, it makes no sense to insist that all types of entropy must have units of thermal entropy—which makes the idea of using thermal entropy increases to compensate for other entropy decreases even less reasonable.

[f)]Dixon has a section "The Entropy Inequality for Open Systems," which contains the inequality, written out in words: "rate of change of entropy inside > rate of entropy flow in - rate of entropy flow out."

people, no doubt because usually the special case of isotropic diffusion (or heat conduction) is assumed, in which case $J = -D\nabla C$ (or $J = -K\nabla T$), and then the numerator in the boundary integral is written as $-D(\partial C/\partial n)$ (or $-K(\partial T/\partial n)$), and in this form it is not obvious that anything is being imported or exported, only that in an isolated system, the boundary integral is zero. Furthermore, entropy as defined by Eq. (A3) or Eq. (A6) is a rather abstract quantity (Mayhew[2] calls it a "mathematical contrivance" and Zhang[13] says it is "not a physical quantity"), and it is hard to visualize what it means to import or export entropy.

Stated in terms of order, Eq. (A8) says that the X-order in a non-isolated system cannot increase faster than it is imported through the boundary [Eq. (A5) says the thermal order cannot increase faster than it is imported]. According to Eq. (A7), the X-order in a system can decrease in two different ways: it can be converted to disorder (first integral term) or it can be exported through the boundary (boundary integral term). It can increase in only one way: by importation through the boundary.

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