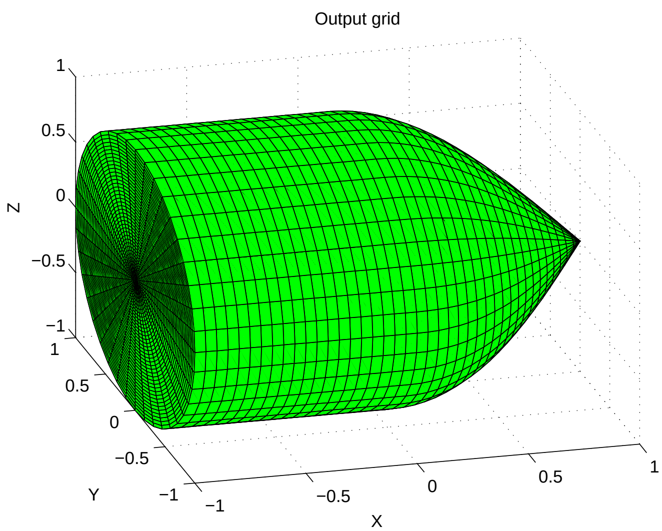
8. a. Now we will solve the steady-state version of the diffusion problem (5.30):

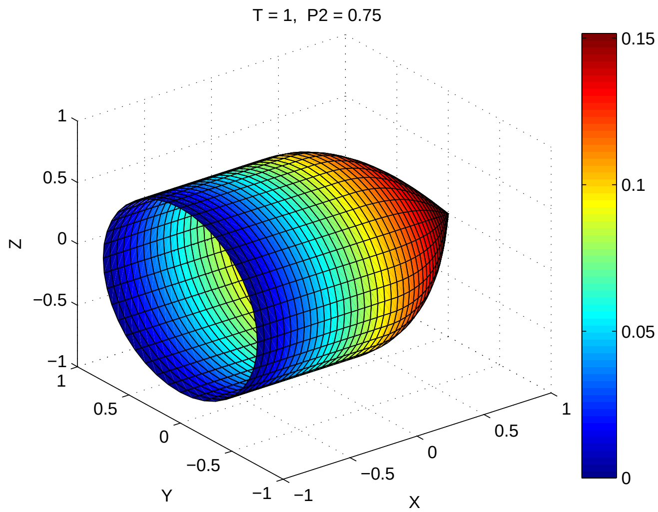
$$0 = \nabla \bullet [D\nabla\mathbf{C}] + q$$

in a 3D bullet-shaped region. The region is a cylinder whose axis is the x-axis, and whose radius varies with x: $r(x) = 1$ for $-1 \leq x \leq 0$ and $r(x) = cos(\pi\ x/2)$ for $0 < x \leq 1$.



We will take the diffusion coefficient $D = 5$, and the source term q will be 1 for $x > 0$ and zero otherwise. Thus, the diffusing element C is being created by a chemical reaction in the forward (nose cone) half of the bullet, but not in the back half. The rear surface is in contact with another material which has no C, so the boundary condition at $x = -1$ will be $C = 0$. The rest of the boundary is insulated, so the normal derivative is zero, $\frac{\partial C}{\partial n} \equiv \nabla C \bullet \mathbf{n} = 0$, that is, the flux is parallel to the boundary. Solve for the concentration C and calculate the integral of C over the entire bullet, and make some MATLAB cross-sectional plots of C. It is important for good accuracy to make sure there is a gridline at $x = 0$, where the source term q is discontinuous.

If you need to define any coefficients, such as q, using an IF statement, remember that you can simply reference a Fortran function and supply the function at the end (see function TRUE(X,Y) in interactive driver example 2). Alternatively, you can add the IF statements when prompted right before the coefficient is used (see D(Z) in interactive driver example 12).



b. If this 3D problem is solved in cylindrical coordinates, that is, if y, z are replaced by polar coordinates r, ϕ, the solution will obviously not depend on the polar angle ϕ, so then 5.30 can be written in the "axi-symmetric" form (cf. formula 5.24b):

$$\frac{\partial C}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(rD\frac{\partial C}{\partial r}\right) + \frac{\partial}{\partial x}\left(D\frac{\partial C}{\partial x}\right) + q$$

Re-solve the problem of part "a" as a 2D steady-state axi-symmetric problem, using the Galerkin method, and again compute the integral of C over the entire bullet (hint: this means you must integrate $2\pi\, r\, C$ over the 2D axi-symmetric cross-section). You will of course have to use "y" for "r", and again it is important that there be no triangles in the initial triangulation (and thus none in the final triangulation) which straddle the interface $x = 0$. On the bottom of the region ($r = 0$) you can use either the boundary condition $\frac{\partial C}{\partial r} = 0$ or "no" boundary condition. Create a MATLAB surface plot of $C(x, y)$.

c. Finally, re-solve the axi-symmetric problem "b" but now with D decreased to 1 in the "nose cone" half of the bullet ($x > 0$) only. Thus the nose cone and the back half of the bullet are made out of different materials, diffusion is slower in the nose cone. Recall that the chemical reaction which is creating C only occurs in the nose cone. It is very easy to treat composite materials such as this using the Galerkin method, one simply defines D to be a discontinuous function of position. Note that $D(x)$ is not constant (even though it is piecewise constant) so you cannot take it out of the brackets in $\frac{\partial}{\partial x}\left[D\frac{\partial C}{\partial x}\right]$. However, $D\frac{\partial C}{\partial x}$ must be continuous, otherwise its derivative would not exist, so $\frac{\partial C}{\partial x}$ must be discontinuous also, see plot of C below. The collocation method cannot handle such problems as easily, because to put this term in the form required by the collocation method, we would have to write as $D\frac{\partial^2 C}{\partial x^2} + \frac{\partial D}{\partial x}\frac{\partial C}{\partial x}$ and $\frac{\partial D}{\partial x}$ would be infinite at $x = 0$.