

# 1D Eigenvalue Problems (Galerkin method)

ODEs (must be linear):

$$\begin{aligned}
 \frac{\partial}{\partial x} A_1(x, U_1, U_{1x}, \dots, U_N, U_{Nx}) &= F_1(x, U_1, U_{1x}, \dots, U_N, U_{Nx}) \\
 &\quad + \lambda \rho_{11}(x) U_1 + \dots + \lambda \rho_{1N}(x) U_N \\
 \cdot &= \cdot \\
 \cdot &= \cdot \\
 \frac{\partial}{\partial x} A_N(x, U_1, U_{1x}, \dots, U_N, U_{Nx}) &= F_N(x, U_1, U_{1x}, \dots, U_N, U_{Nx}) \\
 &\quad + \lambda \rho_{N1}(x) U_1 + \dots + \lambda \rho_{NN}(x) U_N
 \end{aligned}$$

Boundary conditions (at endpoints):

$$\begin{aligned}
 U_1 &= FB_1 \\
 \cdot &= \cdot \\
 \cdot &= \cdot \\
 U_N &= FB_N
 \end{aligned}$$

or ( $N_x = -1$  at left end,  $+1$  at right end)

$$\begin{aligned}
 A_1 N_x &= GB_1(U_1, U_{1x}, \dots, U_N, U_{Nx}) \\
 \cdot &= \cdot \\
 \cdot &= \cdot \\
 A_N N_x &= GB_N(U_1, U_{1x}, \dots, U_N, U_{Nx})
 \end{aligned}$$