1D Time-Dependent Problems (Galerkin method)

PDEs:

$$
\begin{aligned}
C_{11}\frac{\partial U1}{\partial t} + \cdots + C_{1N}\frac{\partial UN}{\partial t} &= \frac{\partial}{\partial x} A_1(x,t,U1,U1_x,...,UN,UN_x) \\
&- F_1(x,t,U1,U1_x,...,UN,UN_x) \\
. &= . \\
. &= . \\
C_{N1}\frac{\partial U1}{\partial t} + \cdots + C_{NN}\frac{\partial UN}{\partial t} &= \frac{\partial}{\partial x} A_N(x,t,U1,U1_x,...,UN,UN_x) \\
&- F_N(x,t,U1,U1_x,...,UN,UN_x)
\end{aligned}
$$

where the C_{ij} are functions of $(x,t,U1,...,UN)$.

Boundary conditions (at endpoints):

$$
\begin{aligned}
U1 &= FB_1(t) \\
. &= . \\
. &= . \\
UN &= FB_N(t)
\end{aligned}
$$

or ($N_x = -1$ at left end, $+1$ at right end)

$$
\begin{aligned}
A_1 N_x &= GB_1(t,U1,U1_x,...,UN,UN_x) \\
. &= . \\
. &= . \\
A_N N_x &= GB_N(t,U1,U1_x,...,UN,UN_x)
\end{aligned}
$$

Initial conditions:

$$
\begin{aligned}
U1(x,t_0) &= U1_0(x) \\
. &= . \\
. &= . \\
UN(x,t_0) &= UN_0(x)
\end{aligned}
$$