2D Eigenvalue Problems (Galerkin method)

PDEs (must be linear):

$$
\begin{aligned}
\frac{\partial}{\partial x} A_1(x, y, U1, U1_x, U1_y, ..., UN, UN_x, UN_y) & \\
+\frac{\partial}{\partial y} B_1(x, y, U1, U1_x, U1_y, ..., UN, UN_x, UN_y) &= \\
F_1(x, y, U1, U1_x, U1_y, ..., UN, UN_x, UN_y) & \\
+\lambda \rho_{11}(x, y) U1 + \cdots + \lambda \rho_{1N}(x, y) UN & \\
. &= \\
. &= \\
\frac{\partial}{\partial x} A_N(x, y, U1, U1_x, U1_y, ..., UN, UN_x, UN_y) & \\
+\frac{\partial}{\partial y} B_N(x, y, U1, U1_x, U1_y, ..., UN, UN_x, UN_y) &= \\
F_N(x, y, U1, U1_x, U1_y, ..., UN, UN_x, UN_y) & \\
+\lambda \rho_{N1}(x, y) U1 + \cdots + \lambda \rho_{NN}(x, y) UN &
\end{aligned}
$$

Boundary conditions:

$$
\begin{aligned}
U1 &= FB_1(x, y) \\
. &= . \\
. &= . \\
UN &= FB_N(x, y)
\end{aligned}
$$

or

$$
\begin{aligned}
A_1 N_x + B_1 N_y &= GB_1(x, y, U1, U1_x, U1_y, ..., UN, UN_x, UN_y) \\
. &= . \\
. &= . \\
A_N N_x + B_N N_y &= GB_N(x, y, U1, U1_x, U1_y, ..., UN, UN_x, UN_y)
\end{aligned}
$$

where (N_x, N_y) = unit outward normal vector.