

3D Eigenvalue Problems

PDEs (must be linear):

$$\begin{aligned}
 F_1(x, y, z, U_1, U_{1x}, U_{1y}, U_{1z}, U_{1xx}, U_{1yy}, U_{1zz}, U_{1xy}, U_{1xz}, U_{1yz}, U_2, \dots) &= \\
 &\lambda\rho_{11}(x, y, z)U_1 + \dots + \lambda\rho_{1N}(x, y, z)U_N \\
 &\quad \cdot \quad = \\
 &\quad \cdot \quad = \\
 F_N(x, y, z, U_1, U_{1x}, U_{1y}, U_{1z}, U_{1xx}, U_{1yy}, U_{1zz}, U_{1xy}, U_{1xz}, U_{1yz}, U_2, \dots) &= \\
 &\lambda\rho_{N1}(x, y, z)U_1 + \dots + \lambda\rho_{NN}(x, y, z)U_N
 \end{aligned}$$

Boundary conditions:

$$\begin{aligned}
 G_1(x, y, z, U_1, U_{1x}, U_{1y}, U_{1z}, \dots, U_N, U_{Nx}, U_{Ny}, U_{Nz}) &= 0 \\
 &\quad \cdot \quad = \cdot \\
 &\quad \cdot \quad = \cdot \\
 G_N(x, y, z, U_1, U_{1x}, U_{1y}, U_{1z}, \dots, U_N, U_{Nx}, U_{Ny}, U_{Nz}) &= 0
 \end{aligned}$$

(Periodic and “no” boundary conditions are also permitted.)