Nov 21 Homework

1. If you want a power series $\sum_{i=0}^{\infty} a_i x^i$ whose coefficients a_i give the number of unordered partitions of i which do not include any 2's, and do not include any parts over 4, what series do you multiply?

2. Write out the first 8 terms (up to $a_7 x^7$) in this series. Actually list the a_7 partitions of 7, using only 1's, 3's and 4's.

3. Find the number of ordered partitions (compositions) of $n = 8$ of p parts, with no part 1. That is, find the number of solutions to $x_1 + x_2 + ... + x_p = 8$, with $x_i \geq 2$. (Hint: substitute $y_i = x_i - 1$.) Clearly, p cannot be more than 4; add up the answers for $p = 1, 2, 3$ and 4, and verify that it is $F_{n-1} = F_7$, as claimed in Schumer problem 15.7.

4. The number of partitions, p(n), for the first 10 integers are:

$$\begin{aligned} p(1) &= 1 \\ p(2) &= 2 \\ p(3) &= 3 \\ p(4) &= 5 \\ p(5) &= 7 \\ p(6) &= 11 \\ p(7) &= 15 \\ p(8) &= 22 \\ p(9) &= 30 \\ p(10) &= 42 \end{aligned}$$

   List all partitions of n, for n=1 through 7.