Solving Quadratic Equations by Completing the Square

Earlier we talked about two special formulas called Perfect Square Trinomials. These formulas are:

$$a^2 + 2ab + b^2 = (a+b)^2$$
$$a^2 - 2ab + b^2 = (a-b)^2$$

Looking at the right side of these formulas, we see an opportunity for the square root property. It would be nice if we could take any quadratic and rewrite it in this form so that the square root property could be used. Many times we can; the process of completing the square is how we do it.

Steps to Solving Quadratic Equations by Completing the Square

1. Isolate the variable terms on one side of the equation.
2. If the coefficient of x^2 is not 1, divide both sides of the equation by the coefficient of x^2 .
3. Take half the coefficient of *x*, then square it. Add this number to both sides of the equation in order to keep it balanced.
4. Factor the quadratic into the square of a binomial: $x^2 + bx = c$ becomes $\left(x + \frac{b}{2}\right)^2 = c + \left(\frac{b}{2}\right)^2$
5. Solve using the square root property.
6. Check your answers in the original equation.

Examples: Solve by completing the square.

1. $x^2 + 8x - 20 = 0$

$x^2 + 8x + 4^2 = 20 + 4^2$

$\frac{1}{2}(8) = 4$ sq

$x^2 + 8x + 16 = 36$

$(x+4)^2 = 36$

$x + 4 = \pm\sqrt{36}$

$x + 4 = \pm 6$

$x + 4 = 6$, -4 -4, $x = 2$

$x + 4 = -6$, -4 -4, $x = -10$

2. $x^2 - 12x + 11 = 0$

$x^2 - 12x + 36 = -11 + 36$

$(-6)^2 = 36$

$x^2 - 12x + 36 = 25$

$(x-6)^2 = 25$

$x - 6 = \pm\sqrt{25}$

$x - 6 = \pm 5$

$x - 6 = 5$, $x = 11$

$x - 6 = -5$, $x = 1$

3. $\frac{2x^2}{2} - \frac{16x}{2} = \frac{4}{2}$

$x^2 - 8x + 16 = 2 + 16$

-4

$(x-4)^2 = 18$

$x - 4 = \pm\sqrt{18}$

$\left\{ \sqrt{18} = \sqrt{9 \cdot 2} = 3\sqrt{2} \right\}$

$x - 4 = +3\sqrt{2}$ $\quad$ $x - 4 = -3\sqrt{2}$

$x = 4 + 3\sqrt{2}$ $\quad$ $x = 4 - 3\sqrt{2}$

4. $4a^2 + 50 = 20a$

$4a^2 - 20a + 50 = 0$

$\frac{4a^2}{4} - \frac{20a}{4} = \frac{-50}{4}$

$a^2 - 5a + 6.25 = -12.5 + 6.25$

-2.5

$(a - 2.5)^2 = -6.25$

~~No~~ No Real solution

5. $\frac{4x^2}{4} - \frac{20x}{4} = \frac{-8}{4}$

$x^2 - 5x + 6.25 = -2 + 6.25$

$(x - 2.5)^2 = 4.25$

$x - 2.5 = \pm\sqrt{4.25}$

$x - 2.5 = \pm\frac{\sqrt{17}}{2}$

$x = 2.5 \pm \frac{\sqrt{17}}{2}$

$x = \frac{5}{2} \pm \frac{\sqrt{17}}{2}$

$4.25 = \frac{17}{4}$

$\sqrt{\frac{17}{4}} = \frac{\sqrt{17}}{\sqrt{4}} = \frac{\sqrt{17}}{2}$