Solving Quadratic Equations by Using the Quadratic Formula

Fact: The solutions to $ax^2 + bx + c = 0$ are given by the quadratic formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Steps to Solving Quadratic Equations Using the Quadratic Formula:

1. Set the quadratic equal to zero.
2. Put the quadratic in standard form.
3. Substitute values for *a, b,* and *c* into the quadratic formula.
4. Simplify the quadratic formula – radical first, then fraction last.
5. Check answers in the original equation.

Examples: Solve using the formula.

1. $4x^2 + 3x - 10 = 0$

$a = 4 \quad b = 3 \quad c = -10$

$x = \dfrac{-(3) \pm \sqrt{(3)^2 - 4(4)(-10)}}{2(4)}$

$x = \dfrac{-3 \pm \sqrt{169}}{8}$

$x = \dfrac{-3 \pm 13}{8}$

$\dfrac{-3+13}{8} = \dfrac{10}{8} = \dfrac{5}{4} = x$

$\dfrac{-3-13}{8} = \dfrac{-16}{8} = -2 = x$

2. $-2h^2 - 7h = -9$

$-2h^2 - 7h + 9 = 0$

$a = -2 \quad b = -7 \quad c = 9$

$h = \dfrac{-(-7) \pm \sqrt{(-7)^2 - 4(-2)(9)}}{2(-2)}$

$h = \dfrac{7 \pm \sqrt{121}}{-4}$

$h = \dfrac{7 \pm 11}{-4}$

$\dfrac{7+11}{-4} = \dfrac{18}{-4} = -\dfrac{9}{2} = h$

$\dfrac{7-11}{-4} = \dfrac{-4}{-4} = 1 = h$

3. $2x^2 + 13x + 15 = 0$

$a = 2 \quad b = 13 \quad c = 15$

$x = \dfrac{-(13) \pm \sqrt{(13)^2 - 4(2)(15)}}{2(2)}$

$x = \dfrac{-13 \pm \sqrt{49}}{4}$

$x = \dfrac{-13 \pm 7}{4}$

$\dfrac{-13+7}{4} = \dfrac{-6}{4} = -\dfrac{3}{2} = x$

$\dfrac{-13-7}{4} = \dfrac{-20}{4} = -5 = x$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

4. $1.5x^2 + 2 = -6.5x$

$1.5x^2 + 6.5x + 2 = 0$

$a = 1.5 \quad b = 6.5 \quad c = 2$

$$x = \frac{-(6.5) \pm \sqrt{(6.5)^2 - 4(1.5)(2)}}{2(1.5)}$$

$$x = \frac{-6.5 \pm \sqrt{30.25}}{3}$$

$$x = \frac{-6.5 \pm 5.5}{3}$$

$$\frac{-6.5 + 5.5}{3} = \frac{-1}{3} = x$$

$$\frac{-6.5 - 5.5}{3} = \frac{-12}{3} = -4 = x$$

Examples: Solve using the square root property, by factoring, or with the quadratic formula.

1. $4x^3 + 5x^2 - 6x = 0$

$x(4x^2 + 5x - 6) = 0$

$x = 0$ or $4x^2 + 5x - 6 = 0$

$a = 4 \quad b = 5 \quad c = -6$

$$x = \frac{-(5) \pm \sqrt{(5)^2 - 4(4)(-6)}}{2(4)}$$

$$x = \frac{-5 \pm \sqrt{121}}{8}$$

$$x = \frac{-5 \pm 11}{8}$$

$$\frac{-5 + 11}{8} = \frac{6}{8} = \frac{3}{4} = x$$

$$\frac{-5 - 11}{8} = \frac{-16}{8} = -2 = x$$

2. $6p^2 + 15 = 21$

$-15 \quad -15$

$$\frac{6p^2}{6} = \frac{6}{6}$$

$p^2 = 1$

$p = \pm\sqrt{1}$ → $p = \pm 1$

$p = -1, 1$

3. $k^2 + 6k = -5$

factor

zero $k^2+6k+5=0$

factor $(k+5)(k+1)=0$

prop. $k+5=0$ or $k+1=0$

$k=-5$ or $k=-1$

Completing Square

$k^2+6k+9=-5+9$

$(k+3)^2=4$

$k+3=\pm\sqrt{4}$

$k+3=\pm 2$

$k+3=2$

$-3\ \ -3$

$k=-1$ ←

$k+3=-2$

$-3\ \ -3$

$k=-5$ ←

4. $1.3x^2 + 4.1x - 7.2 = 0$

decimals → formula

$a=1.3 \quad b=4.1 \quad c=-7.2$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(4.1)\pm\sqrt{(4.1)^2-4(1.3)(-7.2)}}{2(1.3)}$$

$$x=\frac{-4.1\pm\sqrt{54.25}}{2.6}$$

$$\frac{-4.1+\sqrt{54.25}}{2.6}\approx 1.255946127 \Rightarrow x=1.3$$

$$\frac{-4.1-\sqrt{54.25}}{2.6}\approx -4.409792291 \Rightarrow x=-4.4$$