Exponential Functions: Patterns of Growth and Decay

Definition – A basic exponential function can be written in the form $f(x) = a(b)^x$ where *a* and *b* are real numbers $a \neq 0, b > 0$, and $b \neq 1$. The constant *b* is called the base of an exponential function.

Example: The number of ants at a picnic is growing rapidly. At 11:00 AM, 5 ants find the picnic. Each hour after 11:00 AM, 3 times as many ants have found the picnic. Let $A(h)$ represent the number of ants at the picnic *h* hours after 11:00 AM. Start at 11am

a in formula
b in formula

a) Write an equation for a model of $A(h)$

$$A(h) = 5(3)^h$$

b) Estimate numerically when 11,000 ants will be at the picnic.

$A(5) = 1215$
$A(8) = 32805$

about 7 hours later (6pm) there will be nearly 11,000 ants

c) How many ants will be at the picnic at 11:00 PM?

12 hours later
$h = 12$

$$A(12) = 5(3)^{12} = 2{,}657{,}205$$

$y = a(b)^x$

Example: A certain bacteria will double every 15 minutes. If a sample starts with 3 bacteria, find the following.

x2 ??

$a = 3$

a) Find an equation for a model for the number of bacteria after *h* hours.

$B(h) = 3(2)^{4h}$ ← double 4 times every hour

b) Find an equation for a model for the number of bacteria after *n* 15-minute intervals.

$$B(n) = 3(2)^n$$

c) Use your models to estimate the number of bacteria present after 5 hours.

→ 5 hours is $n = 20$ 15-min

$h = 5$

$$B(5) = 3(2)^{(4\times5)} = 3(2)^{20} = 3{,}145{,}728 = B(20) = 3(2)^{20}$$

after (4500 days) time — 1/2 substance remains

Example: An isotope of hydrogen ^3H has a half-life of about 4500 days.

a) Find an equation for a model for the amount of ^3H remaining from a sample of 500 ^3H atoms.

$$H(d) = 500\left(\frac{1}{2}\right)^{d/4500}$$

4500 days to get 1/2

c) Estimate the amount of ^3H remaining after 50 years.

50 yrs ≈ 18,262.5 days

$$H(18262.5) = 500\left(\frac{1}{2}\right)^{\left(\frac{18262.5}{4500}\right)}$$ ≈ 30 atoms remain

Examples: Use the following tables to find exponential models of the given data.

1.

x	0	1	2	3	4
f(x)	20	60	180	540	1620

a = 20, ×3, ×3, ×3

$$\boxed{f(x) = 20(3)^x}$$

2.

x	0	1	2	3	4
f(x)	25	30	36	43.2	51.84

30/25 = 1.2, 1.2, 1.2, 1.2

$$\boxed{f(x) = 25(1.2)^x}$$

3.

x	0	1	2	3	4
f(x)	3200	800	200	50	12.5

÷4

$$\boxed{f(x) = 3200\left(\frac{1}{4}\right)^x}$$

$$y = a(b)^x$$

a = initial value (0, a)

b = base multiplier = amount we multiply by to get next