Exponential Growth and Decay Rates and Compounding Interest

$y = a(b)^x$

Fact – Exponential growth and decay can be modeled by an exponential function where the base multiplier *b* tells us specific information about the growth or decay.

Definition – Growth or Decay Rate – The percentage change in a quantity per 1 unit of time is called the growth or decay rate *r*. The relationship between *r* and *b* is given by $b = 1 + r$.

Example: A swarm of 120 fruit flies in an experiment grows at a rate of about 9.8% per day.

a) Find an equation for a model for the number of fruit flies in the swarm.

$F(x) = 120(1.098)^x$ $\quad x = \#$ days

$b = 1 + 0.098$

$b = 1.098$

b) Estimate the number of fruit flies in the swarm after 20 days.

$F(20) = 120(1.098)^{20} \approx 778$ fruit flies

$y = a(b)^x \quad b = 1 + r$

Example: According to the CIA World Factbook 2008, the population of Liberia can be modeled by $P(t) = 3.1(1.027)^t$, where *P(t)* is the population of Liberia in millions, *t* years since 2005.

a) Use this model to estimate the population of Liberia in 2015.

$P(10) = 3.1(1.027)^{10} = 4.05$

[about 4 million people in 2015]

2015 − 2005 = $t = 10$

b) According to this model, what is the growth rate of Liberia's population?

find r

$b = 1 + r$

$1.027 = 1 + r$

$-1 \quad -1$

$.027 = r$ → 2.7% growth rate

Example: The Gross Domestic Product (GDP) of Madagascar in 2005 was approximately 16.9 billion US$ and has been growing by a rate of about 6% per year.

$b = 1 + r$

$b = 1 + .06 = 1.06$

a) Find an equation for a model for the GDP of Madagascar.

$M(t) = 16.9(1.06)^t$ t since 2005

b) Use your model to estimate the GDP of Madagascar in 2010.

2010 is 5 yrs later...

$M(5) = 16.9(1.06)^5 = \$22.6$ billion

Formula – For compounding interest problems, we use the formula $A = P\left(1+\frac{r}{n}\right)^{(nt)}$, were *A* is the future amount, *P* is the principal amount invested, *r* is the interest rate as a decimal, *t* is time in years, and *n* is the number of compounding periods per year.

quarterly $n = 4$

monthly $n = 12$

Example: If $30,000 is invested in a savings account that pays 4% annual interest compounded daily, what will the account balance be after 6 years?

$P = 30,000$

$r = 0.04$

$n = 365$

$t = 6$

will be => future value = A

$A = 30,000\left(1+\frac{.04}{365}\right)^{(365*6)}$

$A = 38,136.97$

Definition – The number *e* is an irrational number that is $e \approx 2.7182818$.

Fact – When interest is compounded continuously, we use the formula $A = Pe^{(rt)}$.

Example: If the same $30,000 were invested in an account that pays 4% annual interest compounded continuously, how much would you have after 6 years?

$P = 30,000$

$r = 0.04$

$t = 6$

$A = Pe^{(rt)} = 30,000e^{(0.04*6)}$

$= 38,137.47$