Functions and Their Inverses – Intermediate Algebra

My Definition – An inverse function is a function that "undoes" another function.

Fact – The inverse takes any point (x, y) and switches it to the point (y, x). Therefore, the domain of the original is equal to the range of the inverse and the range of the original is equal to the domain of the inverse.

Finding an Inverse Function in a Real-World Problem:

1. Write without function notation.
2. Solve for the "other" variable, that is, the original input variable. This will make it the new output variable.
3. Rewrite in function notation.
4. Caution: Do NOT switch the variables in a real world problem as the variables have meaning and units attached.

Example: To calculate the cost to purchase custom-printed lunch cooler for school staff we use the function $C(L) = 45 + 3L$, where C(L) is the total cost in dollars for purchasing L lunch coolers.

a) What are the input and output variables for this function?

input L = # lunch coolers   Output C = cost of L lunch coolers

b) Find the inverse of this function.

1) $C = 45 + 3L$ 3) $L(C) = \frac{C-45}{3}$

* 2) $C - 45 = 3L$

$\frac{C-45}{3} = L$

c) What are the input and output variables for the inverse?

input is C, cost

output is L lunch coolers

d) Find the cost to purchase 60 lunch coolers.

original $C = 45 + 3(60) = 45 + 180 = \$225$

e) Find the number of lunch coolers you can purchase with a budget of $300.

inverse $L = \frac{300-45}{3} = \frac{255}{3} = 85$ lunch coolers

Example: A team of engineers is trying to pump down the pressure in a vacuum chamber. They know that the following equation represents the pressure in the chamber. $P(s) = 35 - 0.07s$ where $P(s)$ is the pressure in pounds per square inch (psi) of a vacuum chamber after *s* seconds.

a) Find the inverse for this function.

1) $P = 35 - 0.07s$

2) $P - 35 = -0.07s$

$\frac{P-35}{-0.07} = s$

3) $s(P) = \frac{P-35}{-0.07}$

b) Use the inverse function to estimate the time it will take to pump down this vacuum chamber to $\underset{\text{pressure}}{\underline{5 \text{ psi}}}$.

$s(5) = \frac{5-35}{-0.07} = \frac{-30}{-0.07} = 428.57 \approx 429$ seconds

c) If the original function had a domain of $0 \le s \le 500$ and a range of $0 \le P \le 35$, what are the domain and range of the inverse?

Domain inverse (range of original) $0 \le P \le 35$

Range inverse (domain of original) $0 \le s \le 500$

Finding an Inverse Function in a Problem without a Context

1. Write without function notation by replacing *f(x)* with *y*.
2. Solve for *x*, the original input variable. This will make it the new output variable.
3. Interchange the variables *x* and *y*. (All an inverse does is switch input with output.)
4. Rewrite in function notation using $f^{-1}(x)$ to designate it as the inverse.

Examples: Find the inverse of the following functions.

1. $f(x) = 4x - 20$

1) $y = 4x - 20$

2) $y + 20 = 4x$

$\frac{y+20}{4} = x$

3) $y = \frac{x+20}{4}$

4) $\boxed{f^{-1}(x) = \frac{x+20}{4}}$

2. $f(x) = \frac{1}{5}x + 7$

$y = \frac{1}{5}x + 7$

$5(y-7) = \left(\frac{1}{5}x\right)5$

$5(y-7) = x$

$y = 5(x-7) = 5x - 35$

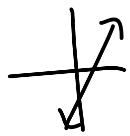
$f^{-1}(x) = 5x - 35$

Definition – A function in which each input corresponds to only one output and each output corresponds to only one input is called a one-to-one function.

Horizontal Line Test (HLT)– If any horizontal line intersects the graph of a function at most once, then that graph is a one-to-one function.

Examples: Graph the following functions using a graphing utility like Desmos. Use the HLT to determine whether each function is a one-to-one function.

a) $f(x) = 2x - 5$

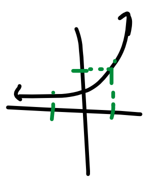


1-1 yes

b) $g(x) = 3(x-2)^2 - 1$

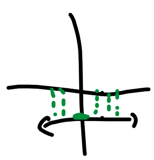
not 1-1

c) $f(x) = 2.6(1.8)^x$



is 1-1

d) $h(x) = -3$



not 1-1

Fact: The graph of a function and its inverse will be symmetric across the line $y = x$. Since the domain and range are interchanged when finding the inverse function, a point (a, b) on the graph of $f(x)$ will correspond to the point (b, a) on the inverse.