Logarithmic Functions – Intermediate Algebra

$b^{\log_b x} = x$

Definition - $y = \log_b(x)$ is the power to which you raise *b* in order to get *x*.

Fact - $y = \log_b(x)$ is the same as $b^y = x$. So all a logarithm is, is an exponent.

Fact – The base most commonly used is base *b = 10* so this is called the common logarithm. It is used so frequently that we do not even write the *b* value. In the sciences, the most natural base to use is $b = e$ so this is called the natural logarithm. As it is so special to the sciences it has a special notation: $\log_e x = \ln x$. Please note that these are "L's" and not "I's" as the word logarithm starts with an L.

$\log_{10} x = \log x$

Examples: Evaluate the following logarithms.

1. $\log_3 9$ → what power do I put on 3 to get 9?

   $\log_3 9 = 2$ because $3^2 = 9$

2. $\log 10,000$

   what power (log) do I put on 10 (no base written) to get 10,000? = 4

3. $\log_5 125$

   $5^? = 125$? = 3 because $5^3 = 125$

4. $\log_2 16$

   $\log_2 16 = 4$ b/c $2^4 = 16$

Properties of Logarithms

Basic 3 prop

1. The logarithm of 1 for any base will always equal zero. $\log_b 1 = 0$ since $b^0 = 1$ $b \neq 0$
2. The logarithm of its base is always equal to 1. $\log_b b = 1$ since $b^1 = b$
3. The logarithm of the base to a power is just that power. $\log_b(b^m) = m$ since $b^m = b^m$.

   what exp do I put on b to get b^m?

Since any base *b* is possible for $b > 0$, it would be impossible to have all of them on the calculator. We can use the change of base formula in order to evaluate any logarithm.

Change of Base Formula - $\log_b a = \dfrac{\log a}{\log b} = \dfrac{\log_c a}{\log_c b}$ for any base *c*.

$$\log_b a = \frac{\ln a}{\ln b}$$

Examples: Evaluate the following logarithms. Round your answers to three decimal places.

my go-to

1. $\log_3 470 = 5.600$

$$\frac{\log(470)}{\log(3)} = 5.600$$

$$\frac{\ln(470)}{\ln(3)} = 5.600$$

2. $\log_7 3.2 = \frac{\ln 3.2}{\ln 7} = 0.598$

3. $\log_{17} 11 = 0.846$

Examples: Rewrite the equation into the opposite form.

1. $6^5 = 7776$

$5 = \log_6 7776$

5 is the power I put on 6 to get 7776.

2. $\log_7 2401 = 4$

$7^4 = 2401$

Fact: To solve a logarithmic equation, we switch to exponential form.

Examples: Solve.

1. $\log x = 4$

$\log_{10} x = 4 \rightarrow 10^4 = x$

$10{,}000 = x$

2. $\log_5 x = 3$

$5^3 = x$

$125 = x$

3. $\log_2 (5x) = 6$

$2^6 = 5x$

$\frac{64}{5} = \frac{5x}{5} \rightarrow x = \frac{64}{5} = 12.8$