

Properties of Logarithms – Intermediate Algebra

Properties of Logarithms:

Product Property - $\log_b(mn) = \log_b m + \log_b n$

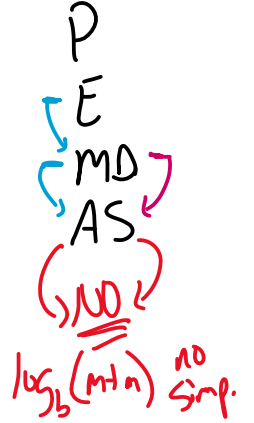
Quotient Property - $\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$

Power Property - $\log_b(m^r) = r \log_b m$

$b^m b^n = b^{m+n}$

$\frac{b^m}{b^n} = b^{m-n}$

$(b^m)^r = b^{mr}$



Examples: Expand the following logarithms as separate simpler logarithms.

1. $\log_7 4a^3b^2 = \log_7 4 + \log_7 a + \log_7 b^2$ (product)
 $= \log_7 4 + 3\log_7 a + 2\log_7 b$ (power)

2. $\log\left(\frac{2x^5}{y^3}\right) = \log(2x^5) - \log(y^3)$ (quot.)
 $= \log(2) + \log(x^5) - \log(y^3)$ (prod.)
 $= \log 2 + 5\log x - 3\log y$ (power)

3. $\log \sqrt{11a^3b^9c} = \log(11a^3b^9c)^{1/2} = \frac{1}{2} \log(11a^3b^9c)$ *

$= \frac{1}{2} (\log(11) + \log(a^3) + \log(b^9) + \log(c))$ (product rule)
 $= \frac{1}{2} (\log(11) + 3\log a + 9\log b + \log c)$ (power rule)
 $= \frac{1}{2} \log(11) + \frac{3}{2} \log a + \frac{9}{2} \log b + \frac{1}{2} \log c$

Big 3 Properties

Prod & Quotient
Finish Power

Remember $\sqrt{u} = u^{1/2}$

Simplify

Start Power rule
End Prod & Quot. rules

Examples: Write the following logarithms as a single logarithm.

$$\begin{aligned}
 1. \log_2 7 + 4\log_2 a + 2\log_2 b & \\
 = \log_2 7 + \log_2 a^4 + \log_2 b^2 & \\
 = \log_2 (7a^4) + \log_2 b^2 & = \boxed{\log_2 (7a^4b^2)}
 \end{aligned}$$

$$\begin{aligned}
 2. 2\log(4x) + 3\log z - 5\log(y) & \\
 = \log(4x)^2 + \log z^3 - \log y^5 & \text{ (power)} \\
 = \log(16x^2) + \log z^3 - \log y^5 & \text{ (prod.)} \\
 = \log(16x^2z^3) - \log y^5 & \\
 \text{Q.R.} & \rightarrow \boxed{\log\left(\frac{16x^2z^3}{y^5}\right)}
 \end{aligned}$$

$(4x)^2 = 4x \cdot 4x = 16x^2$
 simp

$$3. \frac{1}{2}\log_4 5 + \frac{7}{2}\log_4 a - \frac{3}{2}\log_4 c$$

1) Power rule

$$\log_4 5^{1/2} + \log_4 a^{7/2} - \log_4 c^{3/2}$$

2) Product rule

$$\log_4 5^{1/2} a^{7/2} - \log_4 c^{3/2}$$

3) Quotient rule

$$\log_4 \left(\frac{5^{1/2} a^{7/2}}{c^{3/2}} \right) \quad \text{keeper answer}$$

4) Simplification

$$\log_4 \sqrt{\frac{5a^7}{c^3}}$$