Rational Functions – Intermediate Algebra

Definition – A rational expression is an expression of the form $\dfrac{P(x)}{Q(x)}$ where *P(x)* and *Q(x)* are polynomials and $Q(x) \neq 0$. The domain of a rational function consists of those values of *x* such that $Q(x) \neq 0$.

Fact – The domain of a rational function consists of all real numbers EXCEPT those that make the denominator zero.

To Find the Domain of a Rational Function:

1. Focus only on the denominator.
2. Factor the denominator completely.
3. Set the factors NOT equal to zero and solve.

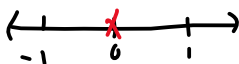Examples: Find the domain of the rational functions.

1. $f(x) = \dfrac{-20}{x}$

   $\boxed{X \neq 0}$

   fancy $\{x \mid x \neq 0\}$ set notation

   interval notation $\boxed{(-\infty, 0), (0, \infty)}$

2. $f(x) = \dfrac{x-6}{x+7}$

   $X + 7 \neq 0$
   $ -7 \quad -7$
   $\rightarrow X \neq -7$ $\boxed{(-\infty, -7), (-7, \infty)}$

3. $f(x) = \dfrac{x+17}{x+9}$

   $X + 9 \neq 0$
   $X \neq -9$ $\boxed{(-\infty, -9), (-9, \infty)}$

4. $f(x) = \dfrac{3x-1}{(2x+1)(x-4)}$

zero
factor
prop.

$(2x+1)(x-4) \neq 0$

$2x+1 \neq 0$

$\dfrac{-1 \quad -1}{2x \neq -1}$

$x \neq \dfrac{-1}{2}$

$x - 4 \neq 0$
$+4 \quad +4$

$x \neq 4$

$\left(-\infty, -\dfrac{1}{2}\right), \left(-\dfrac{1}{2}, 4\right), (4, \infty)$

5. $f(x) = \dfrac{x-3}{(x-3)(5x+11)}$

$(x-3)(5x+11) \neq 0$

$x - 3 \neq 0$ $5x + 11 \neq 0$

$x \neq 3$ $5x \neq -11$

$x \neq -\dfrac{11}{5}$

$\left(-\infty, -\dfrac{11}{5}\right), \left(-\dfrac{11}{5}, 3\right), (3, \infty)$
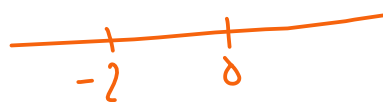
6. $f(x) = \dfrac{3x+6}{x^2+2x}$

$x^2 + 2x \neq 0$

$x(x+2) \neq 0$

$x \neq 0$ $x + 2 \neq 0$

$x \neq -2$

$(-\infty, -2), (-2, 0), (0, \infty)$

7. $f(x) = \dfrac{4x^2 + 3x - 8}{6x^2 - 7x - 20}$

$6x^2 - 7x - 20 \neq 0$

$6x^2 + 8x - 15x - 20 \neq 0$

$2x(3x + 4) - 5(3x + 4) \neq 0$

$(3x + 4)(2x - 5) \neq 0$

$\begin{array}{ll} 3x + 4 \neq 0 & 2x - 5 \neq 0 \\ \underline{-4 \quad -4} & \underline{+5 \quad +5} \\ \dfrac{3x}{3} \neq \dfrac{-4}{3} & \dfrac{2x}{2} \neq \dfrac{5}{2} \\ x \neq -\frac{4}{3} & x \neq \frac{5}{2} \end{array}$

$\begin{array}{cc} & M \quad A \\ 6(-20) = & -120 \quad -7 \\ & 1 \quad -120 \\ & 2 \quad -60 \\ & 3 \quad -40 \\ & 4 \quad -30 \\ & 5 \quad -24 \\ & 6 \quad -20 \quad -14 \\ & 8 \quad -15 \quad -7 \end{array}$

$\boxed{\left(-\infty, -\frac{4}{3}\right), \left(-\frac{4}{3}, \frac{5}{2}\right), \left(\frac{5}{2}, \infty\right)}$

Domain