Adding and Subtracting Rational Expressions – Intermediate Algebra

Finding the Least Common Denominator (LCD) for Rational Expressions

> 1. Factor the denominators (if possible).
> 2. Take the highest power of each factor for the LCD.
> 3. Leave in factored form or multiply the factors together.

Writing Fractions in Terms of the LCD:

> 1. Find the least common denominator.
> 2. Determine what factors of the LCD the fractions' denominator is missing.
> 3. Multiply the numerator and denominator of the fraction by the missing factors.

Examples: Find the LCD and then rewrite each fraction in terms of the LCD.

1. $\dfrac{5}{6x^2}, \dfrac{4}{5x^5}$

$$6x^2 = 2\cdot3x^2$$
$$5x^5 = 5\cdot x^5$$
$$LCD = 2\cdot3\cdot5x^5 = 30x^5$$

$$\frac{5x^3}{5x^3}\cdot\frac{5}{6x^2} = \frac{25x^3}{30x^5}$$

$$\frac{6}{6}\cdot\frac{4}{5x^5} = \frac{24}{30x^5}$$

2. $\dfrac{8p}{12m^2n}, \dfrac{3}{10mn^2}$

$$12m^2n = 3\cdot4\,m^2n = 3\cdot2^2\cdot m^2n$$
$$10mn^2 = 2\cdot5mn^2$$

$$LCD = 3\cdot2^2\cdot5n^2n^2$$
$$= 60m^2n^2$$

$$\frac{5n}{5n}\cdot\frac{8p}{12m^2n} = \frac{40np}{60m^2n^2}$$

$$\frac{6m}{6m}\cdot\frac{3}{10mn^2} = \frac{18m}{60m^2n^2}$$

12
3 4
2 2

3. $\dfrac{a+2}{a-5}, \dfrac{a-3}{a+7}$

$a-5$

$a+7$

$LCD = (a-5)(a+7)$

$\dfrac{(a+7)}{(a+7)} \cdot \dfrac{a+2}{a-5} = \dfrac{(a+7)(a-2)}{(a-5)(a+7)} = \dfrac{(a+7)(a-2)}{LCD}$

$\dfrac{(a-5)}{(a-5)} \cdot \dfrac{(a-3)}{(a+7)} = \dfrac{(a-5)(a-3)}{(a-5)(a+7)}$

4. $\dfrac{2x+5}{x^2+10x+25}, \dfrac{3x-4}{x^2+2x-15}$

$x^2+10x+25 = (x+5)(x+5)$

$x^2+2x-15 = (x+5)(x-3)$

$LCD = (x+5)^2(x-3)$

$\dfrac{x-3}{(x-3)} \cdot \dfrac{2x+5}{(x+5)(x+5)} = \dfrac{(x-3)(2x+5)}{(x+5)^2(x-3)}$

$\dfrac{(x+5)}{(x+5)} \cdot \dfrac{3x-4}{(x+5)(x-3)} = \dfrac{(x+5)(3x-4)}{(x+5)^2(x-3)}$

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To Add Rational Expressions:

$$\text{you must have } LCD$$

1. Find the LCD of all denominators.
2. Rewrite each fraction with the LCD.
3. Combine the numerators using the appropriate operation. The denominator does not change from the LCD when adding or subtracting fractions.
4. Check for simplification possibilities.

Examples: Add.

1. $\dfrac{8}{3a^2} + \dfrac{2}{7a}$ $= \dfrac{56}{21a^2} + \dfrac{6a}{21a^2} = \boxed{\dfrac{56+6a}{21a^2}} =$

$3a^2$
$7a$

$LCD = 3 \cdot 7a^2$
$= 21a^2$

$\dfrac{8}{3a^2} \cdot \dfrac{7}{7} = \dfrac{56}{21a^2}$

$\dfrac{2}{7a} \cdot \dfrac{3a}{3a} = \dfrac{6a}{21a^2}$

2. $\dfrac{h+6}{h+2} + \dfrac{h-5}{h+2}$ $= \dfrac{h+6 + h-5}{h+2} = \boxed{\dfrac{2h+1}{h+2}}$

Same denom ✓

3. $p + \dfrac{3}{p-5}$ $= \dfrac{p^2-5p}{p-5} + \dfrac{3}{p-5} = \boxed{\dfrac{p^2+5p+3}{p-5}} =$

$\dfrac{p}{1}$

$LCD = p-5$

$\dfrac{p}{1} \cdot \dfrac{(p-5)}{p-5} = \dfrac{p^2-5p}{p-5}$

4. $\dfrac{h+14}{(h+5)(h-4)} + \dfrac{h-6}{h-4} \cdot \dfrac{(h+5)}{(h+5)} = \dfrac{h+14}{LCD} + \dfrac{h^2+5h-6h-30}{LCD}$

$LCD = (h+5)(h-4)$

$= \dfrac{h+14+h^2-h-30}{LCD}$

$= \dfrac{h^2-16}{(h+5)(h-4)} = \dfrac{(h+4)(h-4)}{(h+5)(h-4)} = \boxed{\dfrac{h+4}{h+5}}$

5. $\dfrac{a+2}{a^2+5a-6} + \dfrac{a-3}{a^2+8a+12} = \dfrac{a^2+4a+4}{LCD} + \dfrac{a^2+2a-3}{LCD} = \dfrac{2a^2+6a+1}{LCD}$

$a^2+5a-6 = (a+6)(a-1)$
$a^2+8a+12 = (a+6)(a+2)$

$= \boxed{\dfrac{2a^2+6a+1}{(a+6)(a-1)(a+2)}}$

$LCD = (a+6)(a-1)(a+2)$

$\dfrac{a+2}{(a+6)(a-1)} \cdot \dfrac{(a+2)}{(a+2)} = \dfrac{(a+2)(a+2)}{LCD} = \dfrac{a^2+4a+4}{LCD}$

$\dfrac{a+3}{(a+6)(a+2)} \cdot \dfrac{(a-1)}{(a-1)} = \dfrac{(a+3)(a-1)}{LCD} = \dfrac{a^2+2a-3}{LCD}$

Examples: Subtract.

1. $\dfrac{x^2+5x}{(x+3)(x+4)} - \dfrac{3x+8}{(x+3)(x+4)}$ $= \dfrac{x^2+5x-(3x+8)}{LCD}$

$LCD = (x+3)(x+4)$

$= \dfrac{x^2+5x-3x-8}{LCD}$

$= \dfrac{x^2+2x-8}{(x+3)(x+4)} = \dfrac{(x+4)(x-2)}{(x+3)(x+4)} = \boxed{\dfrac{x-2}{x+3}}$

2. $\dfrac{5x+2}{x^2-7x+12} - \dfrac{2x-7}{x^2+4x-21}$ $= \dfrac{5x^2+40x+14}{LCD} - \dfrac{2x^2-15x+28}{LCD}$

$x^2-7x+12 = (x-4)(x-3)$

$x^2+4x-21 = (x+7)(x-3)$

$LCD = (x-4)(x-3)(x+7)$

$= \dfrac{5x^2+40x+14-(2x^2-15x+28)}{LCD}$

$= \dfrac{5x^2+40x+14-2x^2+15x-28}{LCD}$

$\dfrac{5x+2}{(x-4)(x-3)} \cdot \dfrac{(x+7)}{(x+7)} = \dfrac{5x^2+35x+5x+14}{LCD}$

$\dfrac{2x-7}{(x+7)(x-3)} \cdot \dfrac{(x-4)}{(x-4)} = \dfrac{2x^2-8x-7x+28}{LCD}$

$= \boxed{\dfrac{3x^2+55x-14}{(x+7)(x-3)(x-4)}}$

$3x^2+55x-14$

$3(-14)=-42 \mid 55$

$-1 \quad 142 \mid 41$

3. $\dfrac{3x-7}{x^2-4x-45} - \dfrac{2x+3}{x^2-5x-36} = \dfrac{3x^2+5x-28 - (2x^2+13x+15)}{LCD}$

Step 1 factor all denom

$x^2-4x-45 = (x+5)(x-9)$

$x^2-5x-36 = (x+4)(x-9)$

Step 2 Find $LCD = (x+5)(x+4)(x-9)$

Step 3 rewrite with LCD

$\dfrac{3x-7}{(x+5)(x-9)} \cdot \dfrac{(x+4)}{(x+4)} = \dfrac{3x^2+12x-7x-28}{LCD} = \dfrac{3x^2+5x-28}{LCD}$

$\dfrac{2x+3}{(x+4)(x-9)} \cdot \dfrac{(x+5)}{(x+5)} = \dfrac{2x^2+10x+3x+15}{LCD} = \dfrac{2x^2+13x+15}{LCD}$

$= \dfrac{3x^2+5x-28 - 2x^2-13x-15}{LCD}$

$= \boxed{\dfrac{x^2-8x-43}{(x+5)(x+4)(x-9)}}$