

## Solving Rational Equations – Intermediate Algebra

Steps to solve:

1. State what values should be excluded by finding the domain of each expression.
2. Multiply both sides of the equation by the least common denominator.
3. Solve the remaining equation.
4. Check the answer(s) in the original equation.

Examples: Solve the rational equation.

1.  $\frac{35}{x+3} = 5$

$x+3 \neq 0$   
 $x \neq -3$

$$\begin{array}{r} \cancel{(x+3)} \frac{35}{\cancel{x+3}} = 5 \cancel{(x+3)} \\ 35 = 5x + 15 \\ -15 \quad -15 \\ \hline 20 = 5x \end{array}$$

$$\frac{20}{5} = \frac{5x}{5}$$

$$4 = x$$

Check

$$\frac{35}{4+3} = \frac{35}{7} = 5 \checkmark$$

2.  $\frac{8}{x+3} = 7 - \frac{6}{x+3}$

$x+3 \neq 0$   
 $x \neq -3$

$$\begin{array}{r} \cancel{(x+3)} \frac{8}{\cancel{x+3}} = \cancel{(x+3)} 7 - \frac{6}{\cancel{x+3}} \cancel{(x+3)} \\ 8 = 7x + 21 - 6 \end{array}$$

$$\begin{array}{r} 8 = 7x + 15 \\ -15 \quad -15 \\ \hline -7 = 7x \\ \frac{-7}{7} = \frac{7x}{7} \end{array}$$

$$x = -1$$

LHS

$$\frac{8}{-1+3} = \frac{8}{2} = 4$$

RHS

$$7 - \frac{6}{-1+3} = 7 - \frac{6}{2} = 7 - 3 = 4$$

3.  $\frac{4}{x-7} = \frac{8}{x-6}$

$x-7 \neq 0$   
 $x \neq 7$   
 $x-6 \neq 0$   
 $x \neq 6$   
LCD =  $(x-7)(x-6)$

$$\begin{array}{r} \cancel{(x-7)} \cancel{(x-6)} \frac{4}{\cancel{x-7}} = \frac{8}{\cancel{x-6}} \frac{\cancel{(x-7)} \cancel{(x-6)}}{1} \\ 4(x-6) = 8(x-7) \\ 4x - 24 = 8x - 56 \end{array}$$

$$\begin{array}{r} 4x - 24 = 8x - 56 \\ -4x + 56 \quad -4x + 56 \\ \hline 32 = 4x \\ \frac{32}{4} = \frac{4x}{4} \\ 8 = x \end{array}$$

LHS

$$\frac{4}{8-7} = \frac{4}{1} = 4$$

RHS

$$\frac{8}{8-6} = \frac{8}{2} = 4$$

$$4. \frac{3}{a+2} + \frac{5a}{a-3} = \frac{75}{a^2 - a - 6}$$

$$(a-3)(a+2)$$

$$\text{LCD} = (a-3)(a+2) \quad \frac{(a-3)(a+2) \cdot 3}{(a+2)} + \frac{5a \cdot (a-3)(a+2)}{a-3} = \frac{75 \cdot (a-3)(a+2)}{(a-3)(a+2)}$$

$$\rightarrow a-3 \neq 0; a \neq 3$$

$$a+2 \neq 0; a \neq -2$$

$$3(a-3) + 5a(a+2) = 75$$

$$3a - 9 + 5a^2 + 10a = 75$$

$$5a^2 + 13a - 9 = 75$$

$$5a^2 + 13a - 84 = 0$$

$$\begin{array}{r|l} 5(-84) = -420 & 13 \\ -5 & 84 \\ \hline -20 & 21 & 1 \\ -15 & 28 & 13 \end{array}$$

$$5a^2 - 15a + 28a - 84 = 0$$

$$5a(a-3) + 28(a-3) = 0$$

$$(a-3)(5a+28) = 0$$

$$a-3 = 0 \text{ or } 5a+28 = 0$$

$$a = 3$$

extraneous

$$5a = -28$$

$$a = -28/5$$

Example: It takes a garden hose 48 hours to fill a backyard pool. A fire hydrant can fill the same pool in 20 hours. How long will it take to fill the pool if both the garden hose and the fire hydrant are used?

hose 48 hours to fill - can do  $\frac{1}{48}$  of job in 1 hour  
 hydrant 20 hours to fill - can do  $\frac{1}{20}$  of job in 1 hour

Find time working together

t = time together

$$\frac{t}{20} = 4.5$$

$$\frac{t}{48} = 4.12$$

$$\text{LCD} = 4 \cdot 5 \cdot 12 = 240t$$

$$240t \cdot \frac{1}{48} + \frac{1}{20} \cdot 240t = \frac{1}{t} \cdot 240t$$

$$5t + 12t = 240$$

$$17t = 240$$

$$t = \frac{240}{17} = 14.11764706$$

$$14.1 \text{ hours}$$

Example: Jim working alone can paint a house in 30 hours. Mary working alone can paint a house in 24 hours. How long will it take if Jim and Mary work together to paint that house?

Jim 30 hrs 1 job

Mary 24 hrs 1 job

Find  $t$  together

$t$

$$24 = 6 \cdot 4$$

$$30 = 6 \cdot 5$$

$$\text{LCD} = 6 \cdot 4 \cdot 5 \cdot t = 120t$$

$$120t \frac{1}{30} + \frac{120t}{24} = \frac{1}{t} 120t$$

$$4t + 5t = 120$$

$$9t = 120$$

$$t = \frac{120}{9} = 13.\overline{3}$$

13h 20min because

$\frac{1}{3} = .33\overline{3}$  of an hour is 20 min.