Radical Functions – Intermediate Algebra

Definition – A square root or *n*th root is called a radical expression, $\sqrt{x}$ *or* $\sqrt[n]{x}$. The *x* is called the radicand, and *n* is the index. Square roots have an index of 2, but the 2 is not written in the nook of the radical.

Fact – On our calculators we do have a button to give us nth roots (Math, 5: $\sqrt[x]{}$) but we could also use rational exponents. That is $\sqrt[n]{x} = x^{(1/n)}$

Example: Given the function $f(x) = 3.2\sqrt[6]{x}$, find the following.

1. $f(45) = 3.2\sqrt[6]{45} = 6.04$

2. Estimate numerically *x* such that $f(x) = 16$.

$16 = 3.2\sqrt[6]{x}$

$X = 15{,}625$

.

Domain of Radical Functions: n is even

- For even roots, set the radicand greater than or equal to zero to find the domain.
- For odd roots, the domain and range are both all real numbers.

n is odd

Examples: Find the domain and range of the following radical functions.

1. $h(x) = \sqrt[2]{x+10}$

Domain

$x + 10 \geq 0$

$x \geq -10$

$[-10, \infty)$

Range

$y \geq 0$

$[0, \infty)$

Fact $f(x) = \sqrt{x}$

Domain: $[0, \infty)$ ✓

Range: $[0, \infty)$ ✓

2. $g(x)=\sqrt[7]{x+5}$

D: $(-\infty,\infty)$ R: $(-\infty,\infty)$

3. $f(x)=\sqrt[2]{8-x}$

D: $8-x \geq 0$

$-x \geq -8$

$\frac{-x}{-1} \leq \frac{-8}{-1}$

$x \leq 8$ $(-\infty, 8]$

R: $[0,\infty)$

4. $f(x)=\sqrt[2]{3x-2}+5$

D:

$3x-2 \geq 0$

$3x \geq 2$

$x \geq \frac{2}{3}$

$[\frac{2}{3}, \infty)$

R: $[5,\infty)$

5. $f(x)=-\sqrt{x+1}+9$

D: $x+1 \geq 0$

$x \geq -1$

$[-1,\infty)$

Range $(-\infty, 9]$