Solving Radical Equations – Intermediate Algebra

Steps to solving radical equations:

1. Isolate a radical on one side of the equation.
2. "Square" both sides of the equation to eliminate the radical.
3. Isolate any remaining radical if needed and square both sides again.
4. Solve the remaining equation.
5. Check your answer in the original equation. You may have extraneous answers.

Examples: Solve

1. $\sqrt{a}=12$

$(\sqrt{a})^2=(12)^2$

$a=144$

2. $\sqrt{x}=5$

$(\sqrt{x})^2=(5)^2$

$x=25$

3. $\sqrt{m-3}+8=12$

$-8\ \ -8$

$(\sqrt{m-3})^2=(4)^2$

$m-3=16$

$+3\ \ +3$

$m=19$ ✓

4. $\sqrt{x+2}-3=0$

isolate $+3\ +3$

$(\sqrt{x+2})^2=(3)^2$

Square

Solve $x+2=9$

$x=7$

Check: $\sqrt{7+2}-3$

$\sqrt{9}-3$

$3-3=0$ ✓

Example: The period of a simple pendulum for a small amplitude is given by the function $T(L)=2\pi\sqrt{\frac{L}{32}}$ where T(L) is the period in seconds and L is the length of the pendulum in feet.

a) Find the period of a pendulum if its length is 1 foot.

T L=1

$T(1)=2\pi\sqrt{\frac{1}{32}}=2\pi\sqrt{(1/32)}=1.110720735$

$\approx$ 1.11 seconds

b) How long does a pendulum need to be if we want the period to be 2.5 seconds?

Find L T=2.5

$\frac{2.5}{2\pi}=\frac{2\pi\sqrt{\frac{L}{32}}}{2\pi}$

isolate:

$\left(\frac{2.5}{2\pi}\right)^2=\left(\sqrt{\frac{L}{32}}\right)^2$

Square: $32\cdot\left(\frac{2.5}{2\pi}\right)^2=\frac{L}{32}\cdot 32$

$32\left(\frac{2.5}{2\pi}\right)^2=L$

5.066059187

5.07 ft = L

Example: At sea level, the speed of sound through air can be calculated by using the following formula. $c = 340.3\sqrt{\frac{T+273.15}{288.15}}$ where c is the speed of sound in meters per second and T is the temperature in degrees Celsius.

a) Find the speed of sound when the temperature is 25 degrees Celsius.

C $\qquad$ T = 25

$$C = 340.3\sqrt{\frac{(25+273.15)}{288.15}} = 346.15 \text{ m/s}$$

b) Find the temperature if the speed of sound is 350 m/s.

T $\qquad$ C = 350

$$\frac{350}{340.3} = \frac{340.3\sqrt{\frac{T+273.15}{288.15}}}{340.3}$$

isolate:

$$\left(\frac{350}{340.3}\right)^2 = \left(\sqrt{\frac{T+273.15}{288.15}}\right)^2$$

square:

$$(288.15)\left(\frac{350}{340.3}\right)^2 = \frac{T+273.15}{288.15}(288.15)$$

solve:

$$288.15\left(\frac{350}{340.3}\right)^2 = T + 273.15$$

$$-273.15 \qquad -273.15$$

$$288.15\left(\frac{350}{340.3}\right)^2 - 273.15 = T$$

$$31.7^\circ C = T$$

Examples: Solve the following.

1. $\sqrt{a+7} = \sqrt{3a+1}$

Isolate ✓

Square $\left(\sqrt{a+7}\right)^2 = \left(\sqrt{3a+1}\right)^2$

$a+7 = 3a+1$

$-3a-7 \quad -3a-7$

$-2a = -6$

$\frac{-2a}{-2} = \frac{-6}{-2}$

$a = 3$ ✓

2. $\left(\sqrt{4x+3}\right)^2 = \left(7-\sqrt{4x-5}\right)^2$

Isolate ✓

Square $4x+3 = (7-\sqrt{4x-5})(7-\sqrt{4x-5})$

simp $4x+3 = 49 - 7\sqrt{4x-5} - 7\sqrt{4x-5} + 4x - 5$

$4x+3 = -14\sqrt{4x-5} + 4x + 44$

$-4x \quad -4x \quad\quad -4x \quad -44$

Isolate $-41 = -14\sqrt{4x-5}$

$\frac{-41}{-14} = \frac{-14\sqrt{4x-5}}{-14}$

$\left(\frac{41}{14}\right)^2 = \left(\sqrt{4x-5}\right)^2$

Square $\left(\frac{41}{14}\right)^2 = 4x-5$

$+5 \quad\quad +5$

$\left(\frac{41}{14}\right)^2 + 5 = 4x$

$\frac{\left(\frac{41}{14}\right)^2 + 5}{4} = \frac{4x}{4}$

$\frac{\left(\frac{41}{14}\right)^2 + 5}{4} = x$

$3.39 = x$

3. $\sqrt{2x+7} = 3 - \sqrt{x-5}$

Isolate ✓

Square: $\left(\sqrt{2x+7}\right)^2 = \left(3-\sqrt{x-5}\right)^2$ $\quad (3-\sqrt{x-5})$

$2x+7 = 9 - 6\sqrt{x-5} + x - 5$

simp: $2x+7 = 4 + x - 6\sqrt{x-5}$

$-x \quad -4 \qquad -4 \quad -x$

Isolate $(x+3)^2 = \left(-6\sqrt{x-5}\right)^2$

Square $(x+3)(x+3) = (-6\sqrt{x-5})(-6\sqrt{x-5})$

$x^2 + 6x + 9 = 36(x-5)$

Solve:

$x^2 + 6x + 9 = 36x - 180$

$-36x + 180 \quad -36x + 180$

$x^2 - 30x + 189 = 0$

$(x-9)(x-21) = 0$

$x - 9 = 0 \qquad x - 21 = 0$

$x \neq 9 \qquad x \neq 21$

NO solution

m	A
189	-30
-9, -21	-30

$4\left(\sqrt{4x+4}\right)^2 = \left(4+\sqrt{3x-2}\right)^2$

Isolate ✓

Square $4x+4 = (4+\sqrt{3x-2})(4+\sqrt{3x-2})$

Simp $4x+4 = 16 + 8\sqrt{3x-2} + 3x - 2$

$4x+4 = 14 + 3x + 8\sqrt{3x-2}$

$-3x-14 \quad -14 \quad -3x$

$(x-10)^2 = \left(8\sqrt{3x-2}\right)^2$

Isolate & Square $(x-10)(x-10) = 64(3x-2)$

$x^2 - 20x + 100 = 192x - 128$

$-192x + 128 \quad -192x + 128$

$x^2 - 212x + 228 = 0$

$x = \dfrac{-(-212) \pm \sqrt{(-212)^2 - 4(1)(228)}}{2(1)}$

$x = \dfrac{212 \pm \sqrt{43792}}{2}$ $\quad 210.6, \; 1.4$