Absolute Value Equations – Intermediate Algebra

Absolute Value – The absolute value of a real number *n*, $|n|$, is the distance from zero to *n* on a real number line.

Absolute Value Equation – If *n* is a positive real number and *u* is any algebraic expression, then $|u| = n$ means $u = n$ or $u = -n$. If *n* is negative, then the equation has no real solutions. If *n* is zero, $u = 0$.

To solve an absolute value equation:

1. Isolate the expression containing the absolute value.
2. Rewrite the equation as two equations.
3. Solve each equation.
4. Check the solutions in the original absolute value equation.

Examples – Solve the following equations.

1. $|x - 2| = 7$

Isolate ✓
Rewrite $x-2=7$ and $x-2=-7$
$+2\ +2$ $+2\ +2$
$x=9$ and $x=-5$

Check
$x=9$: $|9-2| = |7| = 7$ ✓
$x=-5$: $|-5-2| = |-7| = 7$ ✓

2. $|d - 2| - 9 = -15$

$+9\ +9$
Isolate $|d-2| = -6$ ← negative ⇒ no solution

3. $-3|w + 1| + 12 = -3$

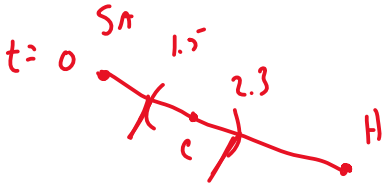
Isolate
$-12\ -12$
$\frac{-3|w+1|}{-3} = \frac{-15}{-3}$
$|w+1| = 5$

$w+1=5$: $-1\ -1$, $w=4$
$w+1=-5$: $-1\ -1$, $w=-6$

$w=4$, $w=-6$

Example – Svetlana is driving from San Antonio to Houston, Texas, on Interstate 10 and will pass through Columbus, Texas, on the way. Svetlana's distance from Columbus can be modeled by the function $D(t) = |125 - 65t|$ where $D(t)$ is Svetlana's distance in miles from Columbus, Texas, after driving for *t* hours. Find the time when Svetlana will be 25 miles away from Columbus.

solve for t D = 25



$$25 = |125 - 65t|$$

$25 = 125 - 65t$ and $-25 = 125 - 65t$

$-125 \quad -125$ $\quad$ $-125 \quad -125$

$-100 = -65t$ $\quad$ $-150 = -65t$

$\frac{-100}{-65} = \frac{-65t}{-65}$ $\quad$ $\frac{-150}{-65} = \frac{-65t}{-65}$

$1.538 = t$ $\quad$ $2.308 = t$