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Rules for Exponents – Intermediate Algebra

We use exponents as a short-hand notation for repeated multiplication. They allow us to write $2x2x2x2x2x2x2 = 2^7$ which is a much more compact form. Every exponential expression has two parts: the base, which is the number repeatedly multiplied, and the exponent, which tells you how many times to multiply.

Product Rule - $x^m x^n = x^{m+n}$ When multiplying exponential expressions that have the same base, add the exponents.

$x^2 \cdot x^3 = (xx)(xxx) = x^5$

Quotient Rule - $\dfrac{x^m}{x^n} = x^{m-n}$ When dividing exponential expressions that have the same base, subtract exponents.

$\dfrac{x^7}{x^4} = \dfrac{xxx\cancel{xxxx}}{\cancel{xxxx}} = x^3$

Power Rule - $\left(x^m\right)^n = x^{mn}$ When raising an exponential expression to another power, multiply the exponents.

$(x^2)^3 = x^2 \cdot x^2 \cdot x^2 = x^6$

Examples: Simplify.

1. $(4w^6x^2)(8wx^9) = 32w^7x^{11}$

   $w^6 \quad w^1$

2. $\dfrac{40t^{11}w^{14}}{5t^3w^9} \Rightarrow \dfrac{40}{5}\,\dfrac{t^{11}}{t^3}\,\dfrac{w^{14}}{w^9} = 8t^{11-3}w^{14-9} = 8t^8w^5$

3. $\dfrac{24b^{18}c^4}{14b^{10}c^3}$

   $\dfrac{24}{14}\,\dfrac{b^{18}}{b^{10}}\,\dfrac{c^4}{c^3}$

   $\dfrac{12}{7}\,b^8c^1$

   $\dfrac{12b^8c}{7} = \dfrac{12}{7}b^8c$

Powers of Products and Quotients – In raising an expression to a power, that power can be applied over multiplication and division. $(xy)^m = x^m y^m$ and $\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$

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Examples: Simplify.

1. $(3x^5y^2z)^3$

$3^3(x^5)^3(y^2)^3(z^1)^3$

$27x^{15}y^6z^3$

2. $\left(\frac{4mp^8}{5m^3p^5}\right)^2 = \frac{4^2m^2(p^8)^2}{5^2(m^3)^2(p^5)^2} = \frac{16m^2p^{16}}{25m^6p^{10}} = \frac{16p^6}{25m^4}$

$\frac{m^2}{m^6} = \frac{1}{m^4}$

has more

$\frac{p^{16}}{p^{10}} = \frac{p^6}{1}$

direction

$\frac{m^2}{m^6} = m^{2-6} = m^{-4} = \frac{1}{m^4}$

3. $(3x^2y^5)(2x^3y)^3$

$(3x^2y^5)(2^3(x^3)^3y^3)$

$(3x^2y^5)(8x^9y^3)$

$24x^{11}y^8$

Negative Exponents - $x^{-n} = \frac{1}{x^n}$ reciprocal **Zero as an Exponent** - $x^0 = 1$ for $x \neq 0$

Rational Exponents - $x^{1/n} = \sqrt[n]{x}$

Examples: Rewrite in radical form

1. $g^{\frac{4}{9}} = g^{4/9} = \sqrt[9]{g^4}$

2. $m^{7/10} = \sqrt[10]{m^7}$

Examples: Simplify

1. $(9a^6b^{10})^0 = 1$

$(\text{house})^0 = 1$

2. $\left(\frac{3h^4}{2p^7}\right)^{-2} = \frac{(3h^4)^{-2}}{(2p^7)^{-2}} = \frac{(2p^7)^2}{(3h^4)^2} = \frac{2^2(p^7)^2}{3^2(h^4)^2} = \frac{4p^{14}}{9h^8}$