Combining Functions – Intermediate Algebra

Definition – A constant, a variable or the product of any number of constants and variables is called a term. Terms can include constants and/or variables raised to exponents.

3, x, $3x^2y^3$, $-7xyz^2$

Definition – The constant part of any term is called the coefficient. The coefficient is usually at the front of any term and includes the sign of the term.

Examples: The 3 in 3x, the -7 in $-7x^2y^3z$.

Definition – Any combination of terms that are added together is called a polynomial. The powers of all variables in a polynomial must be positive integers.

Definition – The sum of all the exponents of the variables in the term is called the degree of a term.

$-7x^2yz^4$ $\quad 2+1+4=7$ is degree

Definition – The degree of the highest term is the degree of the polynomial.

$3x^2+2x-1$ $\quad$ highest deg $=2$

Definition – Like terms are terms that have the same variables with the same exponents.

$3x$, $-21x$ $\quad$ $3x^2$, $-4x^2$, $\quad$ $3x^2$, $4y^2$ not like terms $\rightarrow$ $3x$, $4x^2$

Combining Functions in Applications Using Addition or Subtraction

- The inputs for both functions must be measured in the same units.
- The outputs must be measured in the same units.

Combining Functions in Applications Using Multiplication or Division

- The inputs for both functions must be the same.
- The outputs must make sense together when combined.

Example: Use the following functions to write a new function that will give you the result requested.

F(t) = Number of people employed at Ford Motor Company in year t

I(t) = Average cost, in dollars per employee, for health insurance at Ford Motor Company in year t.

V(t) = Total cost, in dollars, for vacations taken by Ford Motor Company non-management employees in year t.

M(t) = Number of employees of Ford Motor Company who are in management in year t.

a) The total amount spent on health insurance for Ford Motor Company employees in year t.

$I(t) \cdot F(t)$

b) The number of non-management employees at Ford Motor Company in year t.

$F(t) - M(t)$

c) The average cost per non-management employee for vacations at Ford Motor Company in year t.

$\frac{\text{total}}{\text{\#emp}} \qquad \frac{V(t)}{F(t)-M(t)}$

Examples: Combine the following functions. $f(x) = 5x + 6,\ g(x) = 2x - 9,\ h(x) = 3x + 4$

1. $(f+g)(x) = f(x) + g(x)$

$= 5x + 6 + 2x - 9$

$\boxed{(f+g)(x) = 7x - 3}$

2. $(g-f)(x) = g(x) - f(x)$

$= 2x - 9 - (5x + 6)$

$= 2x - 9 - 5x - 6$

$(g-f)(x) = -3x - 15$

3. $(h-g)(x) = h(x) - g(x)$

$= 3x + 4 - (2x - 9)$

$= 3x + 4 - 2x + 9$

$(h-g)(x) = x + 13$

Examples: Given $f(x) = 3x + 8$ and $g(x) = 4x - 10$, find

1. $(fg)(x) = f(x) \cdot g(x)$
$= (3x+8)(4x-10)$
$= 3x(4x) + 3x(-10) + 8(4x) + 8(-10)$
$= 12x^2 - 30x + 32x - 80$
$(fg)(x) = 12x^2 + 2x - 8$

2. $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ $\quad g(x) \neq 0$
$\left(\frac{f}{g}\right)(x) = \frac{3x+8}{4x-10}$
$4x - 10 \neq 0$
$4x \neq 10$
$x \neq \frac{10}{4} \rightarrow x \neq \frac{5}{2}$

Examples: Perform the indicated operations and simplify.

1. $(3m^3n^2 + 5m^2n - 4) + (4m^3n^2 - 2mn - 6)$

$7m^3n^2 + 5m^2n - 2mn - 10$

2. $(8x^3 + 7x^2 - 6x) + (3x^2 + 4x + 7)$

$8x^3 + 7x^2 - 6x - 3x^2 - 4x + 7$
$8x^3 + 4x^2 - 10x + 7$

3. $(3a + 7)(4a - 5)$

$3a(4a) + 3a(-5) + 7(4a) + 7(-5)$
$12a^2 - 15a + 28a - 35 \longrightarrow 12a^2 + 13a - 35$

4. $(5m + 2)(m^2 + 4m - 4)$

$5m(m^2) + 5m(4m) + 5m(-4) + 2(m^2) + 2(4m) + 2(-4)$
$5m^3 + 20m^2 - 20m + 2m^2 + 8m - 8$
$5m^3 + 22m^2 - 12m - 8$