Radicals

If *a* is any non-negative real number, then its square root is the non-negative number whose square is *a*. For example, the square root of 9 is 3 because $3^2 = 9$. We write the square root of *n* as $\sqrt{n}$. (The word root is used interchangeably with the word radical.)

It is important to remember that $\sqrt{n}$ is never negative. Therefore $\sqrt{16}$ is 4 and not -4 even though $(-4)^2 = 16$. If we want to talk about the negative square root of 16 we use the notation $-\sqrt{16} = -4$. If we want both square roots at the same time we use $\pm\sqrt{16} = \pm 4$.

The square root is not the only root possible, it is just the most frequently used root. The cube root of a real number *a* is the number whose cube is *a*. The cube root of *a* is written as $\sqrt[3]{a}$. For example, the cube root of 27 ($\sqrt[3]{27}$) is 3 because $3^3 = 27$. Notice that even though we could only take the square root of a non-negative number, we can take the cube root of any positive number, zero, or negative number. The cube root of a number is always the same sign as the number itself.

Higher roots are defined similarly. The fourth root of a non-negative number *a* is defined as the non-negative number whose fourth power is *a*, and written $\sqrt[4]{a}$. The general *nth* root of any number is $\sqrt[n]{a}$.

NOTE: We cannot take an even-numbered root of a negative number, but we can take an odd-numbered root of any number. Even roots are always positive, whereas odd roots have the same sign as the number we start with.

Radical Rules

If *a* and *b* are any real numbers (non-negative in the case of even-numbered roots), then

$$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

Notice that these rules are similar to the rules for exponents. There are no rules for $\sqrt[n]{a+b}$ or $\sqrt[n]{a-b}$.

Practice Problems: Completely simplify each radical.

1. $\sqrt{4}$
2. $-\sqrt{49}$
3. $\sqrt{\frac{25}{16}}$
4. $\sqrt{8}$
5. $\sqrt{75}$
6. $-\sqrt{\frac{27}{4}}$
7. $\sqrt[3]{54}$
8. $\sqrt[3]{-27x^6y^9}$
9. $\sqrt{\frac{2}{5}}$
10. $-\sqrt{\frac{1}{3}}$
11. $\sqrt[4]{16x^7y^{10}z^{12}}$
12. $\sqrt[3]{-64x^5y^7}$

Solutions

1. 2
2. -7
3. $\frac{5}{4}$
4. $\sqrt{8}=\sqrt{4\cdot 2}=\sqrt{4}\sqrt{2}=2\sqrt{2}$
5. $5\sqrt{3}$
6. $-\frac{3\sqrt{3}}{2}$
7. $3\sqrt[3]{2}$
8. $-3x^2y^3$
9. $\frac{\sqrt{10}}{5}$
10. $-\frac{\sqrt{3}}{3}$
11. $2xy^2z^3\sqrt[4]{x^3y^2}$
12. $-4xy^2\sqrt[3]{x^2y}$