3.2 Compound Interest

Definition – The future value of an investment of PV dollars earning interest at an annual rate of *r* compounded (reinvested) *m* times per year for a period of *t* years is $FV = PV(1+i)^n$, where $i = r/m$ and $n = mt$.

Example: Calculate the FV of an investment of the given amount at the stated interest rate after the stated amount of time. Determine by how much each investment has grown.

1. $8000, at 4% per year, compounded semi-annually, for 8 years.

$i = \frac{.04}{2} = .02$

$n = 2(8) = 16$

$FV = 8000(1+.02)^{16}$

$FV = \$10,982.29$ Grew by $2982.29

2. $16,000, at 2.5% per year, compounded quarterly, for 5 years.

$i = \frac{.025}{4} = 0.00625$

$n = 4(5) = 20$

$FV = 16,000(1+0.00625)^{20}$

$FV = \$18,123.32$ Grew by $2,123.32

3. You try it: $50,000, at 1.5% per year, compounded weekly, for 5 years.

Example: Calculate the PV of an investment that will be worth the given amount at the stated interest rate after the stated amount of time.

1. $7000, after 10 years, at 5% per year compounded monthly

$i = \frac{.05}{12}$

$n = 12(10) = 120$

$7000 = PV\left(1+\frac{.05}{12}\right)^{120}$

$PV = \frac{7000}{\left(1+\frac{.05}{12}\right)^{120}} = \$4250.13$

2. $12,500, after 5 years, at 7% per year compounded daily

$i = \frac{.07}{365}$

$n = 365(5) = 1825$

$PV = \frac{12,500}{\left(1+\frac{.07}{365}\right)^{1825}} = \$8808.90$

Definition – The effective annual interest rate r_{eff} of an investment paying a nominal interest rate of r_{nom} compounded *m* times per year is $r_{eff} = \left(1 + \frac{r_{nom}}{m}\right)^m - 1$. To compare rates of investments with different compounding periods, always compare the effective interest rates rather than the nominal rates.

Examples: Find the effective annual interest rate.

1. 5% compounded quarterly

$$r_{eff} = \left(1 + \frac{.05}{4}\right)^4 - 1 = 0.050945$$

$$r_{eff} = 5.1\%$$

2. 5% compounded monthly

$$r_{eff} = \left(1 + \frac{.05}{12}\right)^{12} - 1 = 0.05116$$

$$r_{eff} = 5.1\%$$

3. You try it: 9% compounded monthly