Chapter Four: Systems of Linear Equations and Matrices

4.1 Systems of Two Equations in Two Unknowns

Definition – A linear equation in two unknowns is an equation that can be written in the form $ax+by=c$ with *a, b,* and *c* being real numbers. The numbers *a* and *b* are called coefficients of their respective variables. A solution of an equation consists of a pair or numbers that when substituted for *x* and *y* yield a true equation.

Graphical Method – To solve a system of two equations in two unknowns we can first solve each equation for *y* and then graph them. The point of intersection will be the solution to the system. Graphing by hand does not always yield an accurate result however, and technology is the preferred graphing method.

Algebraic Method – Multiply each equation by a nonzero number so that the coefficients of *x* are the same value with opposite signs. Add the two equations to eliminate *x*; this gives an equation only in *y* that we can solve to find the value of *y*. Substitute this value back into one of the original equations to find the corresponding *x*. (Note: You could begin by eliminating *y* and solving for *x* first.)

Possible Outcomes for a System of Two Linear Equations in Two Unknowns:

1. A single (unique) solution. This happens when the lines corresponding to the two equations are distinct and not parallel so that they intersect at a single point. This is called a consistent, independent solution.

2. No solution. This happens when the two lines are parallel. We say that the system is inconsistent.

3. An infinite number of solutions. This occurs when the two equations represent the same straight line, and we say that such a system is redundant, or dependent. In this case, we can represent the solutions by choosing one variable arbitrarily and solving for the other.

Examples: Find all solutions of the given systems.

1. $\begin{cases} 2x+3y=5 \\ 3x+2y=5 \end{cases}$ $\quad -3(2x+3y=5)$ $\quad +2(3x+2y=5)$

$$\begin{array}{r} -6x-9y=-15 \\ 6x+4y=10 \\ \hline -5y=-5 \\ y=1 \end{array}$$

$2x+3(1)=5$

$2x+3=5$

$2x=2$

$x=1$

answer: $(1,1)$

2. $\begin{cases} 2x-3y=1 \\ 6x-9y=3 \end{cases}$ multiply by -3 / no need to multiply

$$-6x+9y=-3$$
$$\underline{6x-9y=3}$$
$$0=0$$

$\leftarrow$ always true $\rightarrow$ Solve one equation for y.

$2x-3y=1$

$-3y=1-2x$

$y=\frac{1-2x}{-3}$

Answer: $\left(x, \frac{1-2x}{-3}\right)$

3. $\begin{cases} 2x-3y=2 \\ 6x-9y=3 \end{cases}$ (-3) / good

$$-6x+9y=-6$$
$$\underline{6x-9y=3}$$
$$0=-3$$

$\leftarrow$ never true

answer: no solution

Example: Enormous State University's math department offers two courses: Finite Math and Applied Calculus. Each section of Finite Math has 60 students, and each section of Applied Calculus has 50. The department will offer a total of 110 sections in a semester, and 6000 students would like to take a math class. How many sections of each course should the department offer in order to fill all sections and accommodate all of the students?

1st sentence: two courses FM and AC

2nd sentence: FM has 60 students
AC has 50 students

3rd sentence: total 110 sections
total 6000 students

The question:
How many sections?
(variables are number of sections)

Totals tell equations

Let $x = \#$ sections of FM
$y = \#$ sections of AC

Sections $x+y=110$

Students $60x+50y=6000$

multiply sections by -60:

$$-60x-60y=-6600$$
$$\underline{60x+50y=6000}$$
$$-10y=-600$$
$$y=60 \text{ sections AC}$$

$x+y=110$

$x+60=110$

$x=50$ sections of FM

Example: Anthony is mixing food for his young daughter and would like the meal to supply 1 gram of protein and 5 milligrams of iron. He is mixing together cereal, with 0.5 grams of protein and 1 mg of iron per ounce, and fruit, with 0.2 grams of protein and 2 mg of iron per ounce. What mixture will provide her desired nutrition?

equations $\left\{ \begin{array}{l} \text{totals} \quad 1\text{g protein} \\ \qquad\quad\;\; 5\text{mg iron} \end{array} \right.$

Cereal has per ounce .5g protein, 1mg iron

fruit has per ounce .2g protein, 2mg iron

what mixture – how much of each ingredient. This tells us our variables are ounces of each product

c = ounces of cereal, f = ounces of fruit

protein $.5c + .2f = 1$ mult by -2

iron $1c + 2f = 5$

Solve:

$$\begin{array}{r} -c - .4f = -2 \\ c + 2f = 5 \\ \hline 1.6f = 3 \end{array}$$

$$f = \frac{3}{1.6} = 1.875$$

$c + 2f = 5$

$c + 2(1.875) = 5$

$c + 3.75 = 5$

$c = 1.25$

answer: 1.875 ounces fruit

1.25 ounces cereal

Example: The best sports dorm on campus, the Lombardi House, has won a total of 12 games this semester. Some of these games were soccer games, and the others were football games. According to the rules of the university, each win in a soccer game earns the winning house 2 points, whereas each win in a football game earns them 4 points. If the total number of points Lombardi House earned was 38, how many of each type of game did they win?

Let x = number of soccer games

y = number of football games

1st sentence: $x + y = 12$

points $2x + 4y = 38$

Solve it.