Chapter Seven: Sets and Counting

7.1 Sets and Set Operations.

Definition: A set is a collection of items, referred to as elements of the set.

Fact: We usually use capital letters to represent the sets and lower case letters to indicate the elements. Therefore, *a* is an element of the set A.

Notation:

1. $a \in A$ means that *a* is an element of the set A. If *b* is not an element of the set A, we write $b \notin A$.

2. $B = A$ means that A and B have the same elements. The order in which the elements are listed does not matter.

3. $B \subseteq A$ means that B is a subset of A; every element of B is also in A.

4. $B \subset A$ means that B is a proper subset of A. That is $B \subseteq A$ but $B \neq A$.

5. $\varnothing = \{\ \}$ is the empty set, the set containing no elements. It is a subset of every set.

Fact: A finite set has finitely many elements. An infinite set does not have finitely many elements.

Examples: Some standard sets and what they contain.

1. A coin is {H,T}.

2. Two coins – indistinguishable is {HH, HT, TT} while distinguishable is {HH,HT,TH,TT}.

3. For dice, cards, and three coins, see the handout titled 'dice coin cards.'

In set-builder notation, we do not list each element of the set. For example we may write the set {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} as $\{n | 1 \leq n \leq 10\}$ where the line dividing the set into portions is read as "such that."

Sometimes, visual representations of sets make them easier to deal with. We will use Venn diagrams to represent sets. Please see the 'read it' section of WebAssign to look in the text on pages 397-399 to see examples of various set operations and the accompanying Venn diagrams.

Definition: $A \cup B$ is the union of A and B, the set of all elements that are either in A or in B (or in both). $A \cup B = \{x | x \in A \text{ or } x \in B\}$.

Definition: $A \cap B$ is the intersection of A and B, the set of all elements that are common to A and B.
$A \cap B = \{x | x \in A \text{ and } x \in B\}$.

Definition: If S is the universal set and $A \subseteq S$, then A' is the complement of A (in S), the set of all elements of S that are not in A.

Examples: List the elements of each set.

1. The set F consisting of the four seasons.

$F = \{$Spring, Summer, fall, winter$\}$

2. The set of outcomes of tossing two distinguishable coins.

Penny
dime

$\{HH, HT, TH, TT\}$

3. The set of all outcomes of tossing two indistinguishable dice such that the sum of the numbers is 8.

two blue dice : $\{(2,6), (3,5), (4,4)\}$

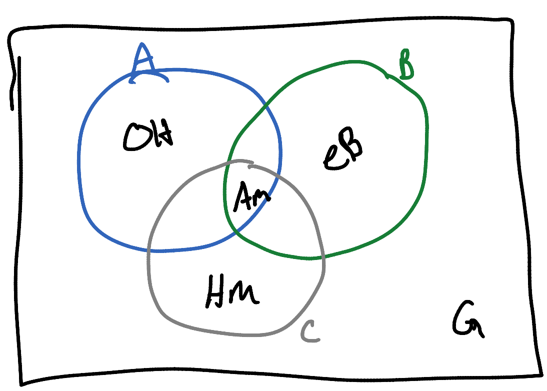
4. The set of all outcomes of tossing two distinguishable dice such that the sum of the numbers is 8.

Green die
red die

$\{(2,6), (3,5), (4,4), (5,3), (6,2)\}$

Examples: Draw a Venn diagram that illustrates the relationships among the given sets. S = {eBay, Google, Amazon, OHaganBooks, Hotmail}. A = {Amazon, OHaganBooks}, B = {eBay, Amazon}, and C = {Amazon, Hotmail}.

Am = Amazon
OH = OHaganBooks
eB = eBay
Hm = Hotmail
G = Google



rectangle give universe S

Circles give sets

Examples: Let A = {June, Janet, Jill, Justin, Jeff, Jello}, B = {Janet, Jello, Justin, Bob}, and C = {Sally, Solly, Molly, Jolly, Jello}. Find each set.

1. $A \cup B$ = {June, Janet, Jill, Justin, Jeff, Jello, Bob}

2. $B \cap C$ = {Jello}

3. $A \cup (B \cup C)$ = {June, Janet, Jill, Justin, Jeff, Jello, Bob, Sally, Solly, Molly, Jolly}

   $B \cup C$ = {Janet, Jello, Justin, Bob, Sally, Solly, Molly, Jolly}

4. $(A \cup B) \cup C$ = All of A, B, and C, with no repeats

   $(A \cup B)$ = all of A and all of B

Definition: The Cartesian Product of two sets, A and B, is the set of all ordered pairs (a,b) with $a \in A$ and $b \in B$. $A \times B = \{(a,b) | a \in A \text{ and } b \in B\}$

Example: Let A = {small, medium, large}, B = {blue, orange}, and C = {triangle, square}.

1. List the elements of $A \times C$

   $A \times C$ = {(sm, blue), (sm, orange), (med, bl), (med, or), (lg, bl), (lg, or)}

   order matters!

2. List the elements of $C \times B$

   $C \times B$ = {($\triangle$, blue), ($\triangle$, orange) ($\square$, blue), ($\square$, orange)}