Chapter Eight: Probability

8.1 Sample Spaces and Events

Definition: An experiment is an occurrence with a result or outcome that is uncertain before the experiment takes place. The set of all possible outcomes is called the sample space for the experiment.

Definition: Given a sample space *S,* an event *E* is a subset of *S*. The outcomes in *E* are called the favorable outcomes. We say that *E* occurs in a particular experiment if the outcome of that experiment is one of the elements of *E* – that is, if the outcome of the experiment is favorable.

Example: Describe the sample space and list the elements of the event.

1. Two coins are tossed; the result is at most one tail.

$$S = \{HH, HT, TH, TT\}$$

$$E = \{HH, HT, TH\} \quad \text{at most 1 is 0 or 1.}$$

2. Three coins are tossed; the result is at most one head.

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

$$E = \{HTT, THT, TTH, TTT\}$$

3. Two distinguishable dice are rolled; the numbers add up to 5.

$$S = 36 \text{ items}$$

$$E = \{(1,4), (2,3), (3,2), (4,1)\}$$

4. Two indistinguishable dice are rolled one of the numbers is even and the other is odd.

$$S = \begin{cases} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), \\ (2,2), (2,3), (2,4), (2,5), (2,6), \\ (3,3), (3,4), (3,5), (3,6), \\ (4,4), (4,5), (4,6), \\ (5,5), (5,6), \\ (6,6) \end{cases}$$

$$E = \{(1,2), (2,3), (1,4), (3,4), (2,5), (4,5), (1,6), (3,6), (5,6)\}$$

5. A letter is chosen at random from the word *Mozart*; the letter is a vowel.

$$S = \{M, o, z, a, r, t\} \qquad E = \{o, a\}$$

Definition: The complement of an event E is the set of outcomes not in E. Thus, the complement of E represents the event that E does not occur.

Definition: The union of two events E and F is the set of all outcomes in E or F (or both). Thus $E \cup F$ represents the event that E occurs or F occurs or both.

Definition: The intersection of the events E and F is the set of all outcomes common to E and F. Thus $E \cap F$ represents the event that both E and F occur.

Definition: If E and F are events, then E and F are said to be disjoint or mutually exclusive if $E \cap F$ is empty. (Hence, they have no outcomes in common.)

Fact: It is impossible for mutually exclusive events to occur simultaneously.

Example: The table shows the performance of a selection of 100 stocks after one year. Take S to be the set of all stocks represented in the table.

	Pharmaceutical (P)	Electronic (E)	Internet (I)	Total
Increased (V)	10	5	15	30
Unchanged (N)	30	0	10	40
Decreased (D)	10	5	15	30
Total	50	10	40	100

1. Use symbols to describe the event that a stock's value increased but it was not an Internet stock. How many elements are in this event?

$$V \cap I' \qquad n(V \cap I') = 15$$

2. Use symbols to describe the event that an Internet stock did not increase. How many elements are in this event?

$$I \cap V' \qquad n(I \cap V') = 25$$

3. Compute $n(P' \cup N)$. What does this number represent?

P' = not Pharmaceutical
N = unchanged
U = all stocks that are not pharm or are unchanged or both

$$n(P' \cup N) = n(P') + n(N) - n(P' \cap N)$$
$$= 50 + 40 - 10$$
$$= 80$$

4. Compute $n(P \cup N')$. What does this number represent?

all stocks that are pharm or changed or both.

$$n(P \cup N') = n(P) + n(N') - n(P \cap N')$$
$$= 50 + 60 - 20$$
$$= 90$$

5. Find all pairs of mutually exclusive events among the events P, E, I, V, N, and D.

P + E   V + N
P + I   V + D
E + I   N + D

6. Find all pairs of events that are not mutually exclusive among the events P, E, I, V, N, and D.

P + V   E + V   I + V
P + N   E + N   I + N
P + D   E + D   I + D

7. Calculate $\dfrac{n(V \cap I)}{n(I)}$. What does this answer represent?

$$\frac{15}{40} = \frac{3}{8}$$

This is the ratio of increasing internet stocks to all internet stocks.

8. Calculate $\dfrac{n(D \cap I)}{n(D)}$. What does this answer represent?

$$\frac{15}{30} = \frac{1}{2}$$

This is the ratio of decreasing internet stocks to all decreasing stocks.

Example: A bag contains six gummy bears. Noel picks four at random. How many possible outcomes are there? If one of the gummy bears is raspberry, how many of these outcomes include the raspberry gummy bear?

have = 6

want = 4

$C(6,4) = 15$

raspberry: $C(1,1) \cdot C(5,3) = 1 \cdot 10 = 10$