8.3 Probability and Probability Models

Theoretical Probability

Intuitive Definition: The probability, *P(E),* of an event *E* is the fractions of times we expect *E* to occur if we repeat the experiment over and over.

Math Definition: The probability, *P(E),* of an event *E* is the limiting value of the estimated probability as the number of trials gets larger and larger. That is, the estimated probability approaches the theoretical probability as the number of trials gets larger and larger.

Computing Theoretical Probability for Equally Likely Outcomes – In an experiment in which all outcomes are equally likely, the theoretical probability of an event *E* is given by

$$P(E)=\frac{Number\ of\ \ favorable\ outcomes}{Total\ number\ of\ outcomes}=\frac{n(E)}{n(S)}$$

Examples: Compute the theoretical probability, given that all outcomes are equally likely.

1. $n(E)=13,\ n(S)=20$

$P(E)=\frac{n(E)}{n(S)}=\frac{13}{20}$

2. $n(S)=40,\ n(E)=22$

$P(E)=\frac{n(E)}{n(S)}=\frac{22}{40}=\frac{11}{20}$

Examples: Find the probability of each event, assuming the coins are fair and the dice are distinguishable and fair.

1. Two coins are tossed; the result is at most one tail.

$n(S)=4$ $n(E)=3$

$P(E)=\frac{3}{4}$

2. Three coins are tossed; the result is at most one head.

$n(S)=8$ $n(E)=4$

$P(E)=\frac{4}{8}=\frac{1}{2}$

3. Two dice are rolled; the numbers add up to 5.

$n(S)=36$ $n(E)=4$

$P(E)=\frac{4}{36}=\frac{1}{9}$

4. Two dice are rolled; the numbers add up to 1.

$n(S)=36$ $n(E)=0$

$P(E)=\frac{0}{36}=0$ (this is an impossible event)

5. Two dice are rolled; both numbers are prime.

$n(S)=36$ $n(E)=9$

2,2 3,2 5,2
2,3 3,3 5,3
2,5 3,5 5,5

$P(E)=\frac{9}{36}=\frac{1}{4}$

Definition: If *A* and *B* are any two events, then $P(A\cup B)=P(A)+P(B)-P(A\cap B)$. The reason for the subtraction has to do with the intersection being part of both *A* and *B.* If *A* and *B* are mutually exclusive events, $P(A\cup B)=P(A)+P(B)$ because there is no intersection.

Principles of Probability Distributions (Part 2)

1. P(S) = 1 as the probability of something happening is 1.
2. $P(\varnothing)=0$ as the probability of nothing happening is 0.
3. $P(A')+P(A)=1$ as the sample space must consist of everything in an event plus everything not in that event.

Example: Find the indicated probability.

$$P(A)=0.1, P(B)=0.6, P(A\cap B)=0.05. Find\ P(A\cup B)$$

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$
$$=0.1+0.6-0.05$$
$$=0.65$$