2.2 Basic Differentiation Rules and Rates of Change

The Constant Rule – The derivative of a constant function is 0. That is, if *c* is a real number, then $\frac{d}{dx}[c]=0$.

The Power Rule – If *n* is a rational number, then the function $f(x)=x^n$ is differentiable and $\frac{d}{dx}\left[x^n\right]=nx^{n-1}$. For *f* to be differentiable at *x = 0*, *n* must be a number such that x^{n-1} is defined on an interval containing 0.

Examples: Find the slope of the tangent line at the given value.

1. $f(x)=x^5,\ x=2$

   Using the power rule, $f'(x)=5x^{5-1}=5x^4$.

   $m=f'(2)=5(2)^4=5(16)=80$

2. $f(x)=x^{2/3},\ x=1$

   Power rule: $f'(x)=\frac{2}{3}x^{\frac{2}{3}-1}=\frac{2}{3}x^{-1/3}$

   $m=f'(1)=\frac{2}{3}(1)^{-1/3}=\frac{2}{3}$

3. $f(x)=\frac{1}{x^3},\ x=-2$

   simplify/rewrite first: $f(x)=\frac{1}{x^3}=x^{-3}$

   power rule: $f'(x)=-3x^{-3-1}=-3x^{-4}=-\frac{3}{x^4}$

   $m=f'(-2)=\frac{-3}{(-2)^4}=\frac{-3}{16}$

The Constant Multiple Rule – If *f* is a differentiable function and *c* is a real number, then *cf* is also differentiable and $\frac{d}{dx}\left[cf(x)\right]=cf'(x)$.

Examples: Find the derivative.

1. $f(x)=3x^5$ $\quad f'(x)=3(5x^{5-1})=15x^4$

2. $f(x)=17x^2$ $\quad f'(x)=17(2x^{2-1})=34x$

3. $f(x)=-5x^{-7}$ $\quad f'(x)=-5(-7x^{-7-1})=35x^{-8}$

The Sum and Difference Rules – The sum (or difference) of two differentiable functions *f* and *g* is itself differentiable. Moreover, the derivative of *f* + *g* (or *f* – *g)* is the sum (or difference) of the derivatives of *f* and *g*.

$$\frac{d}{dx}\left[f(x)+g(x)\right]=f'(x)+g'(x)$$
$$\frac{d}{dx}\left[f(x)-g(x)\right]=f'(x)-g'(x)$$

With constant multiplies and sums or difference, we can now find the derivative of any polynomial or root function using the rules rather than limits.

Examples: Use the rules of differentiation to find the derivative of the function.

1. $f(x)=-9$ $\quad$ Constant function so $f'(x)=0$

2. $f(x)=\sqrt[4]{x}$

$f(x)=x^{1/4}$ $\quad$ so $\quad f'(x)=\frac{1}{4}x^{\frac{1}{4}-1}=\frac{1}{4}x^{-3/4}$

3. $f(x) = x^2 - 2x + 3$

$f'(x) = 2x^{2-1} - 2(1x^{1-1}) + 0 = 2x - 2$

4. $y = 8 - x^3$

$y' = 0 - 3x^{3-1} = -3x^2$

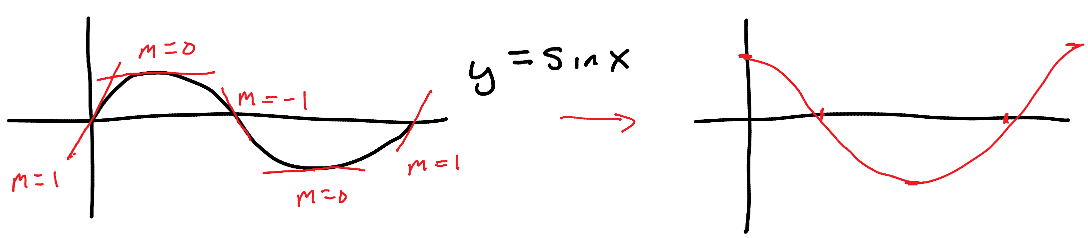
5. $f(x) = 2x^3 - x^2 + 3x$

$f'(x) = 2(3x^{3-1}) - 2x^{2-1} + 3(1x^{1-1})$
$= 6x^2 - 2x + 3$

Notice that the derivative of $y = mx$ is always m.
(why? $y = mx$ so $y' = m(1x^{1-1}) = mx^0 = m$.)

Derivatives of the Sine and Cosine Functions – By observing the graphs of these two basic trig functions we find that

$$\frac{d}{dx}[\sin x] = \cos x \qquad \frac{d}{dx}[\cos x] = -\sin x$$



slopes give the graph of $y' = \cos x$

you can do a similar analysis for $y = \cos x$ and $y' = -\sin x$.

Examples: Use the rules of differentiation to find the derivative of the function.

1. $g(t) = \pi \cos t$ $\qquad g'(t) = \pi(-\sin t) = -\pi \sin t$

Get in the habit of simplifying derivatives.

2. $y = 7 + \sin x$

$y' = 0 + \cos x = \cos x$

3. $y = \dfrac{5}{(2x)^3} + 2\cos x \;=\; \dfrac{5}{8x^3} + 2\cos x \;=\; \dfrac{5}{8}x^{-3} + 2\cos x$ (Rewrite)

$y' = \frac{5}{8}(-3x^{-3-1}) + 2(-\sin x) = -\frac{15}{8}x^{-4} - 2\sin x = \dfrac{-15}{8x^4} - 2\sin x$

Examples: Find the slope of the graph of the function at the given point.

slope ⇒ derivative

1. $f(x) = \dfrac{8}{x^2}$, $(2,2)$

$f(x) = 8x^{-2}$ so $f'(x) = 8(-2x^{-2-1}) = -16x^{-3} = \dfrac{-16}{x^3}$

At $(2,2)$, $f'(2) = \dfrac{-16}{(2)^3} = \dfrac{-16}{8} = -2$

2. $f(t) = 3 - \dfrac{3}{5t}$, $\left(\dfrac{3}{5}, 2\right)$ $\qquad f(t) = 3 - \frac{3}{5}t^{-1}$

$f'(t) = 0 - \frac{3}{5}(-1t^{-1-1}) = 0 + \frac{3}{5}t^{-2} = \dfrac{3}{5t^2}$

$f'\left(\frac{3}{5}\right) = \dfrac{3}{5\left(\frac{3}{5}\right)^2} = \dfrac{3}{5\left(\frac{9}{25}\right)} = \dfrac{3}{\frac{9}{5}} = \dfrac{\cancel{3}}{1} \cdot \dfrac{5}{\cancel{9}_3} = \dfrac{5}{3}$

divide by $\frac{9}{5}$
is mult by $5/9$

3. $f(\theta) = 4\sin\theta - \theta, (0,0)$

$f'(\theta) = 4(\cos\theta) - 1\theta^{1-1} = 4\cos\theta - 1$

$f'(0) = 4\cos 0 - 1 = 4(1) - 1 = 4 - 1 = 3$

Examples: Find the derivative.

1. $f(x) = \dfrac{x^3 - 6}{x^2}$ No quotient rule yet so first we simplify

$f(x) = \dfrac{x^3}{x^2} - \dfrac{6}{x^2} = x - \dfrac{6}{x^2} = x - 6x^{-2}$

$f'(x) = 1 - 6(-2x^{-2-1}) = 1 + 12x^{-3} = 1 + \dfrac{12}{x^3}$

2. $y = 3x(6x - 5x^2)$ no product rule yet so simplify/rewrite

$y = 18x^2 - 15x^3$

$y' = 18(2x^{2-1}) - 15(3x^{3-1}) = 36x - 45x^2$

3. $y = \dfrac{2}{\sqrt[3]{x}} + 3\cos x = 2x^{-1/3} + 3\cos x$

$y' = 2(-\frac{1}{3}x^{-\frac{1}{3}-1}) + 3(-\sin x) = -\frac{2}{3}x^{-4/3} - 3\sin x$

Example: Determine the point(s) at which the graph of $f(x) = x + \sin x,\ 0 \le x < 2\pi$ has a horizontal tangent line. we are looking for points where $f'(x)=0$ $m=0$

$f'(x) = 1 + \cos x$

$0 = 1 + \cos x$

$-1 = \cos x$ on $0 \le x < 2\pi$

$\pi = x$

Find the point (x, y) with $x = \pi$

$y = f(\pi) = \pi + \sin\pi = \pi + 0 = \pi$

$f(x) = x + \sin x$ has a horizontal tangent line at the point (π, π)

Example: The number of gallons *N* of regular unleaded gasoline sold by a gasoline station at a price of *p* dollars per gallon is given by *N = f(p)*.

a) Describe the meaning of *f'(2.979)*

The derivative has units of N divide by units of p so $f'(2.979)$ is the rate of change of number of gallons sold divide by price per gallon when $p = \$2.979/\text{gallon}$

b) Is *f'(2.979)* usually positive or negative? Explain.

Usually this value is negative as stations will sell less fuel at higher prices. (The rate at which they sell fuel is negative.)