3.6 A Summary of Curve Sketching

Guidelines for Analyzing the Graph of a Function

1. Determine the domain and range of the function.

2. Determine the intercepts, asymptotes, and symmetry of the graph.

3. Locate the x-values for which *f'*(x) and *f"(x)* either are either zero or do not exist. Use the results to determine relative extrema and points of inflection.

Example: Analyze and sketch the graph.

1. $y = -2x^4 + 3x^2$

$y = x^2(-2x^2+3)$

X-Int = Zeros: $0 = x^2(3-2x^2)$

$x = 0 \qquad 3 - 2x^2 = 0$

$3 = 2x^2$

$\frac{3}{2} = x^2$

$x = \pm\sqrt{\frac{3}{2}}$

$(0,0)$

$(-\frac{\sqrt{6}}{2}, 0)$

$(\frac{\sqrt{6}}{2}, 0)$

y-int at $(0,0)$

no asymptotes

$y' = -8x^3 + 6x$

$0 = -2x(4x^2-3)$

$x = 0 \qquad 4x^2 - 3 = 0$

$4x^2 = 3$

$x^2 = \frac{3}{4}$

$x = \pm\frac{\sqrt{3}}{2}$

$-1 \quad -\frac{1}{2} \quad \frac{1}{2} \quad 1$

$+ \quad -\frac{\sqrt{3}}{2} \quad - \quad 0 \quad - \quad \frac{\sqrt{3}}{2} \quad +$

inc max dec dec min inc

$y'' = -24x^2 + 6$

$0 = -24x^2 + 6$

$24x^2 = 6$

$x^2 = \frac{6}{24}$

$x^2 = \frac{1}{4}$

$x = \pm\frac{1}{2}$

$-1 \quad 0 \quad 1$

$- \quad -\frac{1}{2} \quad + \quad \frac{1}{2} \quad -$

cc down ↑ cc up ↑ cc down

p.o.i. p.o.i.

We found the locations of some important points so now we find the corresponding y-values:

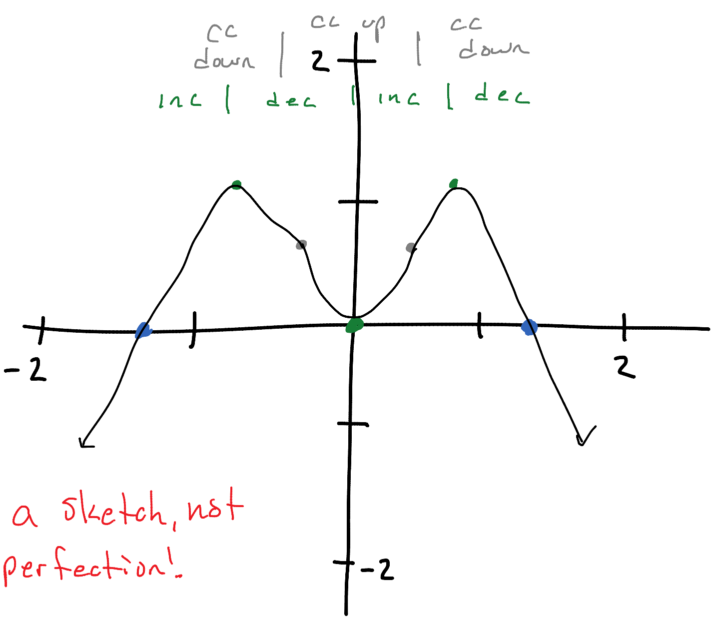
max $y(-\frac{\sqrt{3}}{2}) = -2(-\frac{\sqrt{3}}{2})^4 + 3(-\frac{\sqrt{3}}{2})^2 = \frac{9}{8}$

$y(0) = 0$

min $y(\frac{\sqrt{3}}{2}) = \frac{9}{8}$

points of inflection $y(-\frac{1}{2}) = \frac{5}{8}$

$y(\frac{1}{2}) = \frac{5}{8}$



a sketch, not perfection!

2. $f(x)=\dfrac{2x^2-5x+5}{x-2}$

$f(x)\neq 0$ so no x-int.

Vert. asy. $x=2$

y-int $\left(0,\frac{5}{-2}\right)=\left(0,-\frac{5}{2}\right)$

No horiz. asy as $\lim\limits_{x\to\infty} f(x)$ dne

$$f'(x)=\frac{(x-2)(4x-5)-(2x^2-5x+5)(1)}{(x-2)^2}$$

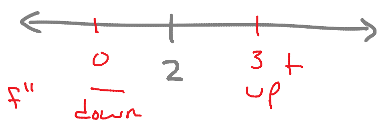
$$f'(x)=\frac{4x^2-5x-8x+10-2x^2+5x-5}{(x-2)^2}$$

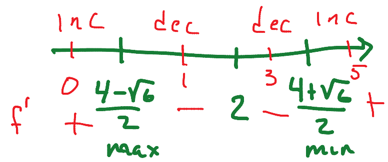
$$f'(x)=\frac{2x^2-8x+5}{(x-2)^2}$$

$f'(x)=0$ when

$x=\frac{4\pm\sqrt{6}}{2}$ (approx 3.2 and 0.8)

$f'(x)$ dne at $x=2$





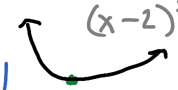
$$f''(x)=\frac{(x-2)^2(4x-8)-(2x^2-8x+5)2(x-2)}{(x-2)^4}=\frac{(x-2)(4x-8)-2(2x^2-8x+5)}{(x-2)^3}$$

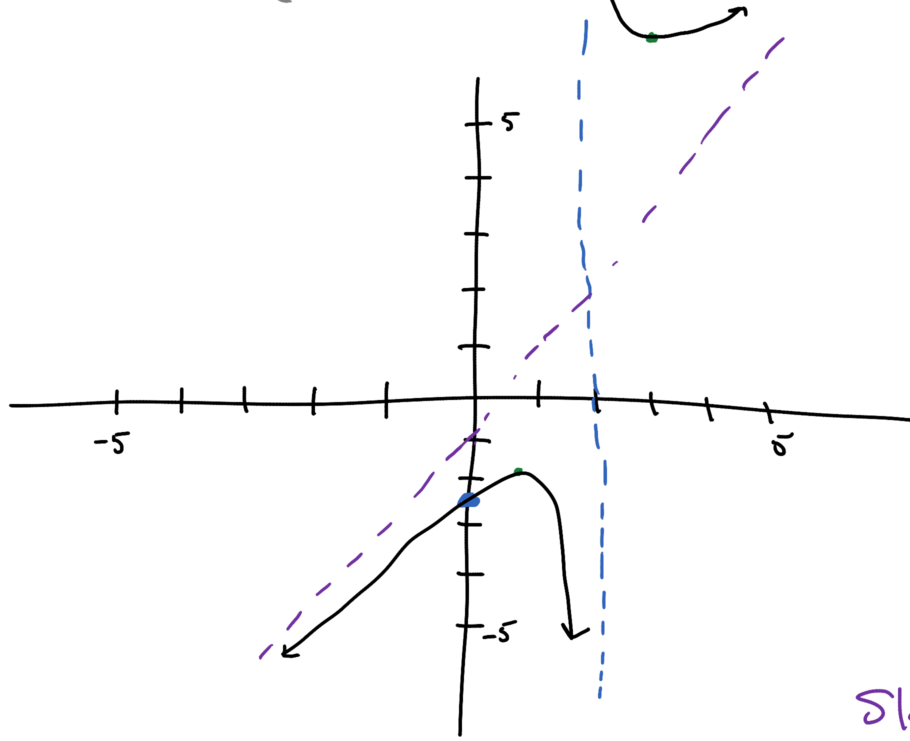
$$f''(x)=\frac{4x^2-8x-8x+16-4x^2+16x-10}{(x-2)^3}=\frac{6}{(x-2)^3}$$

$f''(x)\neq 0$

$f''(x)$ dne at $x=2$

not a point of inflection





points

$\left(\frac{4-\sqrt{6}}{2},\ 3-2\sqrt{6}\right)$ max

$(0.8,-1.9)$

$\left(\frac{4+\sqrt{6}}{2},\ 3+2\sqrt{6}\right)$ min

$(3.2,7.9)$

Slant asymptote

2	2	-5	5
		4	2
	2	-1	3

$y=2x-1$

3. $y = x\sqrt{9-x^2}$

domain: $9-x^2 \geq 0$

$(3-x)(3+x) \geq 0$

$-4 \quad 0 \quad 4$

$- \ \ -3 \ \ + \ \ 3 \ \ -$

so domain is $[-3, 3]$

zeros at $x = -3, 0, 3$

y-int at $(0,0)$

$$y' = x\tfrac{1}{2}(9-x^2)^{-1/2}(-2x) + \sqrt{9-x^2}$$

$$y' = \frac{-x^2}{\sqrt{9-x^2}} + \frac{\sqrt{9-x^2}}{1}\cdot\frac{\sqrt{9-x^2}}{\sqrt{9-x^2}} = \frac{-x^2}{\sqrt{9-x^2}} + \frac{9-x^2}{\sqrt{9-x^2}} = \frac{9-2x^2}{\sqrt{9-x^2}}$$

$y' = 0$ when $9-2x^2 = 0$ so $9 = 2x^2$ or $\frac{9}{2} = x^2$

this is $\pm\sqrt{\frac{9}{2}} = x = \frac{\pm 3}{\sqrt{2}} = \pm\frac{3\sqrt{2}}{2}$ (approx ± 2.12)

$-2.5 \quad 0 \quad 2.5$

$-3 \quad -\frac{3\sqrt{2}}{2} \quad \frac{3\sqrt{2}}{2} \quad 3$

$-$ dec, min, $+$ inc, max, $-$ dec

$$y'' = \frac{\sqrt{9-x^2}(-4x) - (9-2x^2)\tfrac{1}{2}(9-x^2)^{-1/2}(-2x)}{(\sqrt{9-x^2})^2}$$

$$y'' = \frac{-4x\sqrt{9-x^2}\cdot\frac{\sqrt{9-x^2}}{\sqrt{9-x^2}} + \frac{\cancel{2}x(9-2x^2)}{\cancel{2}\sqrt{9-x^2}}}{9-x^2} = \frac{\frac{-4x(9-x^2)}{\sqrt{9-x^2}} + \frac{x(9-2x^2)}{\sqrt{9-x^2}}}{9-x^2} = \frac{-36x+4x^3+9x-2x^3}{(9-x^2)^{3/2}}$$

$$y'' = \frac{2x^3-27x}{(9-x^2)^{3/2}}$$

$y'' = 0$ when $2x^3 - 27x = 0$ so $x(2x^2-27) = 0$

or $x = 0$ and $2x^2 - 27 = 0$

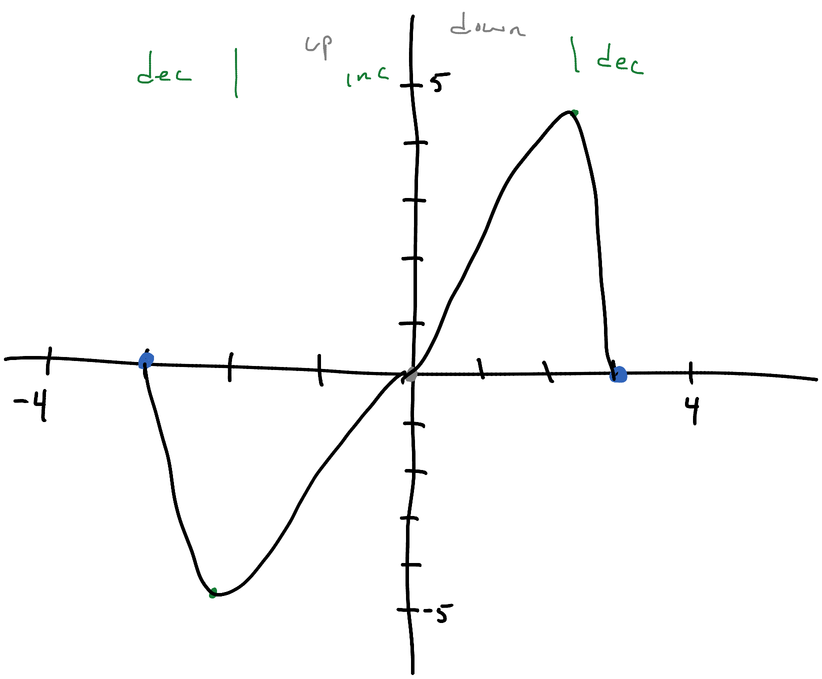
$2x^2 = 27 \to x^2 = \frac{27}{2} \to x = \pm\sqrt{\frac{27}{2}} = \pm\frac{3\sqrt{6}}{2}$

out of domain (approx ± 3.7)

$-1 \quad 1$

$-3 \quad + \quad 0 \quad - \quad 3$

up, down



points:

$\left(\frac{-3\sqrt{2}}{2}, -\frac{9}{2}\right)$ min

$\left(\frac{3\sqrt{2}}{2}, \frac{9}{2}\right)$ max

$(0,0)$ point of inflection

$(-3,0)$ x-intercepts

$(0,0)$

$(3,0)$