Math 1508

Extra Practice Handout

Trigonometry Big 3

1. Find the amplitude, period, phase shift, and five key points for each function.

a) $y=\sin\left(x+\frac{\pi}{2}\right)$

$A=|1|=1$

$P=\frac{2\pi}{1}=2\pi$

P.S. $x+\frac{\pi}{2}=0 \Rightarrow x=-\frac{\pi}{2}$

Normal sine pts: $(0,0)\ (\frac{\pi}{2},1)\ (\pi,0)\ (\frac{3\pi}{2},-1)\ (2\pi,0)$

Shifted left $\pi/2$:

$(-\frac{\pi}{2},0), (0,1), (\frac{\pi}{2},0), (\pi,-1), (\frac{3\pi}{2},0)$

b) $y=3+\frac{1}{2}\cos\left(2x-\frac{\pi}{2}\right)$

$A=|\frac{1}{2}|=\frac{1}{2}$

$P=\frac{2\pi}{2}=\pi$

P.S. $2x-\frac{\pi}{2}=0$

$\frac{1}{2}\cdot 2x=\frac{\pi}{2}\cdot\frac{1}{2}$

$x=\frac{\pi}{4}$

Normal cosine points: $(0,1),(\frac{\pi}{2},0),(\pi,-1),(\frac{3\pi}{2},0),(2\pi,1)$

1st $x=\frac{\pi}{4}$ (p.s.)

1st $y=3+\frac{1}{2}(1)=3.5$

2nd $x \Rightarrow 2x-\frac{\pi}{2}=\frac{\pi}{2}$

$x=\frac{\pi}{2}$

2nd $y=3+\frac{1}{2}(0)=3$

3rd $x \Rightarrow 2x-\frac{\pi}{2}=\pi$

$x=3\pi/4$

$y=3+\frac{1}{2}(-1)=2.5$

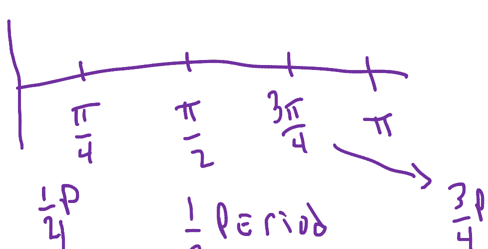
4th $x=\pi$, $y=3$

5th $x=\frac{5\pi}{4}$, $y=3.5$

points: $(\pi/4,3.5),(\frac{\pi}{2},3),(\frac{3\pi}{4},2.5),(\pi,3),(\frac{5\pi}{4},3.5)$

c) $y=\cos 2x$

$A=1$ $\quad P=\frac{2\pi}{2}=\pi$ $\quad$ p.s. $2x=0$ so $x=0$



Points: $(0,1), (\frac{\pi}{4},0), (\frac{\pi}{2},-1), (\frac{3\pi}{4},0)$

$(\pi,1)$

d) $y=\frac{1}{2}\sin x+\underline{1}$ $A=\frac{1}{2}$ $P=2\pi$ p.s. $x=0$

v.t.

points $(0,1)$, $(\frac{\pi}{2},1.5)$, $(\pi,1)$, $(\frac{3\pi}{2},\frac{1}{2})$, $(2\pi,1)$

$y=\frac{1}{2}(0)+1$ $y=\frac{1}{2}(1)+1$ $y=\frac{1}{2}(0)+1$ $y=\frac{1}{2}(-1)+1$ $y=\frac{1}{2}(0)+1$

e) $y=-2\sin\frac{1}{2}x-3$ $A=|-2|=2$ $P=\frac{2\pi}{\frac{1}{2}}=4\pi$ p.s. $x=0$

p.s. $\frac{1}{4}P$ $\frac{1}{2}P$ $\frac{3}{4}P$ P

points: $(0,-3)$, $(\pi,-5)$, $(2\pi,-3)$, $(3\pi,-1)$, $(4\pi,-3)$

$y=-2(0)-3$ $y=-2(1)-3$ $y=-2(0)-3$ $y=-2(-1)-3$ $y=-2(0)-3$

f) $y=-\cos\left(x-\frac{\pi}{2}\right)$ $A=|-1|=1$ $P=2\pi$ p.s. $x-\frac{\pi}{2}=0$

$x=\frac{\pi}{2}$

points: $(\frac{\pi}{2},-1)$, $(\pi,0)$, $(\frac{3\pi}{2},1)$, $(2\pi,0)$, $(\frac{5\pi}{2},-1)$

$y=-(1)$ $y=-(0)$ $y=-(-1)$ $y=-(0)$ $y=-(1)$

2. Verify the identity.

$\sin^2 x + \cos^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

a) $\dfrac{1-\sin x}{\cos x} = \dfrac{\cos x}{1+\sin x}$

$$LHS = \frac{1-\sin x}{\cos x} \cdot \frac{1+\sin x}{1+\sin x} = \frac{1-\sin^2 x}{\cos x(1+\sin x)} = \frac{\cos^2 x}{\cos x(1+\sin x)} = \frac{\cos x}{1+\sin x} = RHS$$

b) $\dfrac{1+\cos 2\theta}{\sin 2\theta} = \cot\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

$\sin 2\theta = 2\sin\theta\cos\theta$

$$LHS = \frac{1+\,?}{2\sin\theta\cos\theta} = \frac{1+2\cos^2\theta-1}{2\sin\theta\cos\theta} = \frac{2\cos^2\theta}{2\sin\theta\cos\theta} = \frac{\cos\theta}{\sin\theta} = \cot\theta = RHS$$

GOAL: $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$, NEED TO CANCEL $\cos\theta$

c) $\dfrac{\tan\gamma + \sin\gamma}{2\tan\gamma} = \cos^2\gamma$

$$LHS = \frac{\tan y}{2\tan y} + \frac{\sin y}{2\tan y} = \frac{1}{2} + \frac{\sin y}{2\left(\frac{\sin y}{\cos y}\right)} = \frac{1}{2} + \frac{\cancel{\sin y}}{2}\cdot\frac{\cos y}{\cancel{\sin y}}$$

$$= \frac{1}{2} + \frac{\cos y}{2} = \frac{1+\cos y}{2} = \text{WHAT NOW?!?}$$

Formula sheet: $\cos^2 y = \dfrac{1+\cos 2y}{2}$

This is not an identity!

If we had $RHS = \cos^2\left(\frac{y}{2}\right)$ it would be!

d) $\dfrac{\sin x-\cos x}{\cos^2 x}=\dfrac{\tan^2 x-1}{\sin x+\cos x}$

RHS LOOKS more complicated, BUT I DON'T KNOW what to do with it. SO...

$$\text{LHS}=\frac{\sin x-\cos x}{\cos^2 x}\cdot\frac{\sin x+\cos x}{\sin x+\cos x}=\frac{\sin^2 x-\cos^2 x}{\cos^2 x(\sin x+\cos x)}=\frac{\sin^2 x}{\cos^2 x(\sin x+\cos x)}-\frac{\cos^2 x}{\text{DENOM}}$$

$$=\frac{\tan^2 x}{\sin x+\cos x}-\frac{1}{\sin x+\cos x}=\frac{\tan^2 x-1}{\sin x+\cos x}$$

3. Solve in the interval $[0,2\pi)$.

a) $4\sin^2 x=1$

$$\sqrt{\sin^2 x}=\sqrt{\tfrac{1}{4}}$$

$$\sin x=\pm\frac{1}{2}$$

ALL 4 QUADS

$\text{ref}=\frac{\pi}{6}$

$$x=\frac{\pi}{6},\ \frac{5\pi}{6},\ \frac{7\pi}{6},\ \frac{11\pi}{6}$$

b) $\underline{\sin 2x}\sin x-\cos x=0$

$$2\sin x\cos x\sin x-\cos x=0$$

$$\cos x(2\sin^2 x-1)=0$$

$$\cos x=0 \qquad 2\sin^2 x-1=0$$

$$x=\frac{\pi}{2},\ \frac{3\pi}{2}$$

$$2\sin^2 x=1$$

$$\sin^2 x=\frac{1}{2}$$

$$\sin x=\pm\sqrt{\tfrac{1}{2}}=\pm\frac{\sqrt{2}}{2}$$

ALL 4 Q

$\frac{\pi}{4}$ ref

$$x=\frac{\pi}{4},\ \frac{3\pi}{4},\ \frac{5\pi}{4},\ \frac{7\pi}{4}$$

c) $2\cos^2 x + 3\cos x = -1$

$2\cos^2 x + 3\cos x + 1 = 0$

$(2\cos x + 1)(\cos x + 1) = 0$

$2\cos x + 1 = 0$ or $\cos x + 1 = 0$

$2\cos x = -1$ $\qquad$ $\cos x = -1$

$\cos x = -\frac{1}{2}$ $\qquad$ $\cancel{x = \frac{3\pi}{2}}$ ← No! $\cos x = -1$ AT $x = \pi$

II + III

ref $\frac{\pi}{3}$

$\cos x = -\frac{1}{2}$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$

d) $\sin^2 x - 7\sin x = 0$

$\sin x(\sin x - 7) = 0$

$\sin x = 0$ or $\sin x - 7 = 0$

$x = 0, \pi, \cancel{2\pi}$ $\qquad$ $\sin x = 7$

NOT IN interval

* impossible, range of sine is [-1, 1]

e) $\csc^2 x - 2\cot^2 x = 0$

$1 + \cot^2 x - 2\cot^2 x = 0$

$1 - \cot^2 x = 0$

$1 = \cot^2 x$

$\pm 1 = \cot x$

All 4 $\qquad$ $\pi/4$ ref

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

f) $\sin 4x + 2\sin 2x = 0$

$4x = 2(2x)$

$2\sin(2x)\cos(2x) + 2\sin 2x = 0$

$2\sin(2x)(\cos 2x + 1) = 0$

$2\sin(2x) = 0$

$\sin(2x) = 0$

$\cos 2x + 1 = 0$

$\cos 2x = -1$

* x is in $[0, 2\pi)$ But 2x is in $[0, 4\pi)$

$\sin(2x) = 0$ when $2x = 0, \pi, 2\pi, 3\pi, 4\pi$

So $x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$, $\cancel{2\pi}$

$\cos 2x = -1$ when $2x = \cancel{\frac{3\pi}{2}, \frac{7\pi}{2}}$ Nope... my bad!

So $x = \cancel{\frac{3\pi}{4}, \frac{7\pi}{4}}$

$2x = \pi, 3\pi$

so $x = \frac{\pi}{2}, \frac{3\pi}{2}$