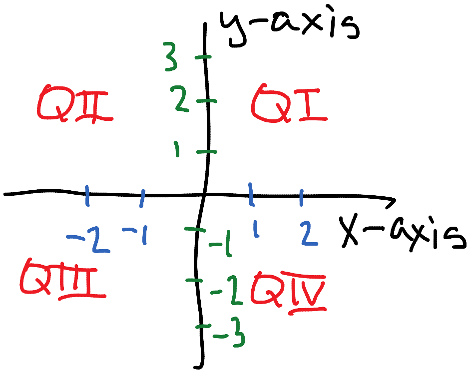
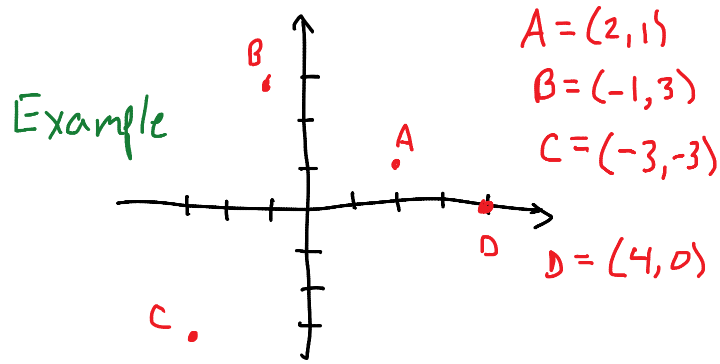
Chapter One – Functions and Their Graphs

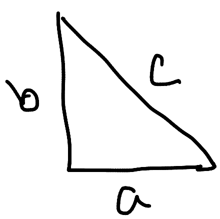
1.1 Rectangular Coordinates

The Cartesian coordinate system consists of two number lines that meet at the 0 points at a 90 degree angle. The point at which the zero points meet is called the origin and divides the plane into four regions called quadrants. The horizontal line (axis) is known as the x-axis and the vertical line (axis) is known as the y-axis. Using these as guide values, we can name any point in any quadrant as (x,y).

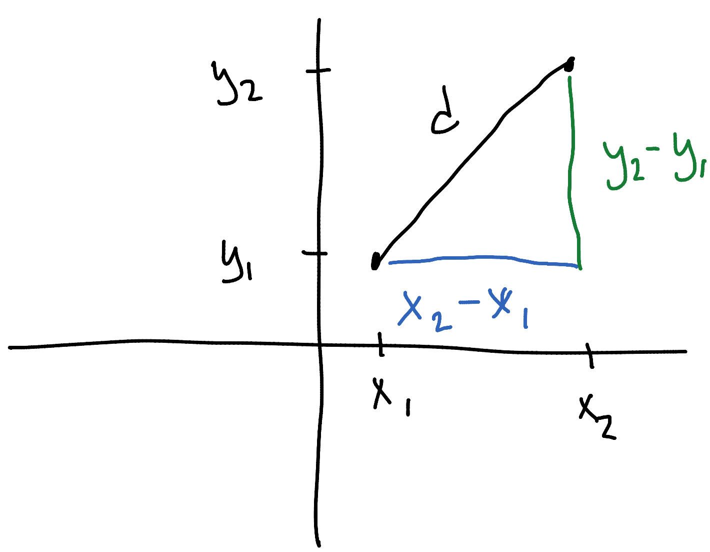




Pythagorean Theorem – For a right triangle with hypotenuse length *c* and sides of lengths *a* and *b*, you have $a^2 + b^2 = c^2$. The converse is also true. That is, if the equation holds then the triangle must be a right triangle.



With the help of the Pythagorean Theorem and our coordinate system, we have the distance formula.



Distance Formula – The distance *d* between the points (x_1, y_1) and (x_2, y_2) in the plane is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Example: Find the distance between the points.

1. (8,5) and (20,0)

$$d = \sqrt{(20-8)^2 + (0-5)^2} = \sqrt{12^2 + (-5)^2} = \sqrt{144+25} = \sqrt{169}$$
$$= 13$$

2. (-2,6) and (3,-4)

$$d = \sqrt{(3-(-2))^2 + (-4-6)^2} = \sqrt{(3+2)^2 + (-10)^2} = \sqrt{25+100} = \sqrt{125}$$

Simplify: $\sqrt{125} = \sqrt{25 \cdot 5} = \sqrt{25}\sqrt{5} = \boxed{5\sqrt{5}}$

Example: Show that the points form the vertices of a right triangle. We must find AB, AC, BC

1. (4,0), (2,1), and (-1,-5)
A B C

$$d_{AB} = \sqrt{(2-4)^2 + (1-0)^2} = \sqrt{(-2)^2 + (1)^2} = \sqrt{4+1} = \sqrt{5}$$

$$d_{AC} = \sqrt{(-1-4)^2 + (-5-0)^2} = \sqrt{(-5)^2 + (-5)^2} = \sqrt{25+25} = \sqrt{50}$$

$$d_{BC} = \sqrt{(-1-2)^2 + (-5-1)^2} = \sqrt{(-3)^2 + (-6)^2} = \sqrt{9+36} = \sqrt{45}.$$

We need $a^2+b^2=c^2$. We know the hypotenuse is the longest side (always) of the triangle so

$(\sqrt{5})^2 + (\sqrt{45})^2 = (\sqrt{50})^2 \rightarrow$ it is a right triangle

2. (-1,3), (3,5), and (5,1)
A B C

$$d_{AB} = \sqrt{(3-(-1))^2 + (5-3)^2} = \sqrt{4^2 + 2^2} = \sqrt{16+4} = \sqrt{20}$$

$$d_{AC} = \sqrt{(5-(-1))^2 + (1-3)^2} = \sqrt{6^2 + (-2)^2} = \sqrt{36+4} = \sqrt{40}$$

$$d_{BC} = \sqrt{(5-3)^2 + (1-5)^2} = \sqrt{2^2 + (-4)^2} = \sqrt{4+16} = \sqrt{20}$$

These could be simplified but it wouldn't help

$(\sqrt{20})^2 + (\sqrt{20})^2 = (\sqrt{40})^2$ so is a right triangle.

The Midpoint Formula – The midpoint of the line segment joining the points (x_1, y_1) and (x_2, y_2) is given by the midpoint formula $mp = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$. Note: The midpoint formula comes from the distance formula. A proof can be found at the end of chapter one of the text and is available from me if requested.

Examples: Find the midpoint of the following sets of points.

1. (8,13) and (16, -5)

$$mp = \left(\frac{8+16}{2}, \frac{13+{}^-5}{2}\right) = \left(\frac{24}{2}, \frac{8}{2}\right) = (12,4)$$

2. (-1, 3) and (4, -10)

$$mp = \left(\frac{-1+4}{2}, \frac{3+{}^-10}{2}\right) = \left(\frac{3}{2}, \frac{-7}{2}\right)$$

Get used to fractions, Many times they are better than the decimal representation