

1.2 Graphs of Equations

A relationship between two quantities is frequently expressed as an equation in two variables. These equations have one or more solutions, or solution points. The set of all solutions of an equation in two variables represents the graph of the equation.

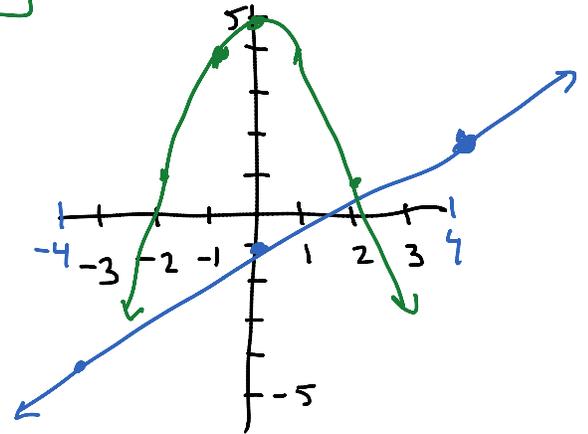
Sketching the Graph of an Equation by Point Plotting:

1. If possible, rewrite the equation so that one of the variables is isolated on one side of the equation.
2. Make a table of values showing several solution points.
3. Plot these points on a rectangular coordinate system.
4. Connect the points with a smooth curve or line.

Examples: Sketch the graph of $y = \frac{3}{4}x - 1$ and then sketch $y = 5 - x^2$.

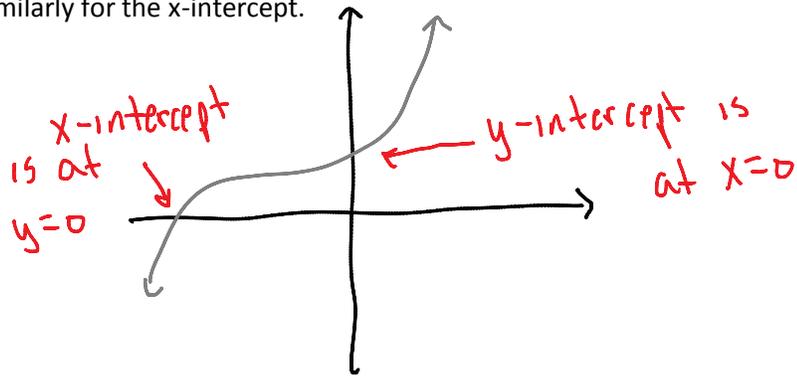
x	$y = \frac{3}{4}x - 1$
-4	$\frac{3}{4}(-4) - 1 = -4$
0	$\frac{3}{4}(0) - 1 = -1$
4	$\frac{3}{4}(4) - 1 = 2$

x	$y = 5 - x^2$
-2	$5 - (-2)^2 = 1$
-1	$5 - (-1)^2 = 4$
0	$5 - (0)^2 = 5$
1	$5 - (1)^2 = 4$
2	$5 - (2)^2 = 1$



Choose any
x, so choose
easy x!

Intercepts occur when a graph crosses, or touches, an axis. Naturally, we refer to the graph crossing the y-axis as the y-intercept, and similarly for the x-intercept.



Finding Intercepts:

1. To find x-intercepts, let y be zero and solve the equation for x .
2. To find y-intercepts, let x be zero and solve the equation for y .

Symmetry of a graph is helpful to know before graphing so that you only have to find half the points. Symmetry is like a mirror in that one area of the graph is mirroring another area. The key is to find the mirror, or line of symmetry.

Graphical Tests for Symmetry:

Blue

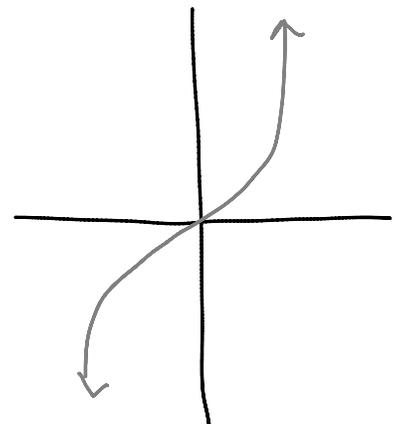
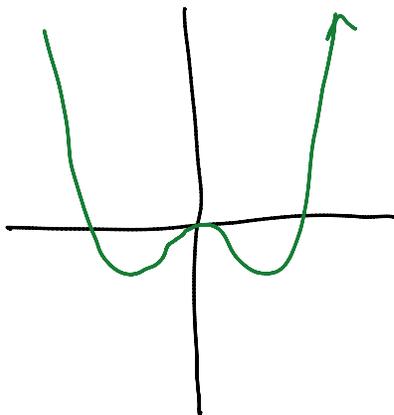
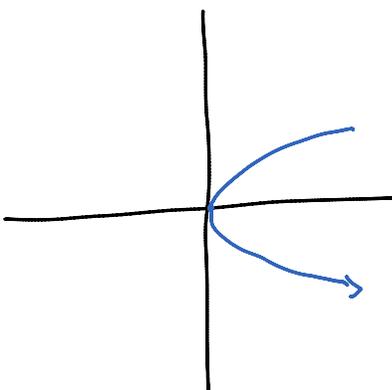
1. A graph is symmetric with respect to the x-axis if, whenever (x,y) is on the graph, $(x,-y)$ is also on the graph.

Green

2. A graph is symmetric with respect to the y-axis if, whenever (x,y) is on the graph, $(-x,y)$ is also on the graph.

gray

3. A graph is symmetric with respect to the origin if, whenever (x,y) is on the graph, $(-x,-y)$ is also on the graph.



Algebraic Tests for Symmetry:

1. The graph of an equation is symmetric with respect to the x-axis if replacing y with $-y$ yields an equivalent equation.
2. The graph of an equation is symmetric with respect to the y-axis if replacing x with $-x$ yields an equivalent equation.
3. The graph of an equation is symmetric with respect to the origin if replacing x with $-x$ and y with $-y$ yields an equivalent equation.

A function is called even if it is symmetric with respect to the y-axis. A function is called odd if it is symmetric with respect to the origin.

Examples: Find the intercepts of the graph of the equation.

1. $y = 8 - 3x$

y-int has $x=0$:
 $y = 8 - 3(0) = 8$

$(0, 8)$

x-int has $y=0$

$$0 = 8 - 3x$$

$$3x = 8$$

$$x = 8/3$$

$(\frac{8}{3}, 0)$

2. $y = \sqrt{2x-1}$

y-int
 $y = \sqrt{2(0)-1}$
 $y = \sqrt{-1}$
not a real number

no y-int.

x-int

$$0 = \sqrt{2x-1}$$

$$0^2 = (\sqrt{2x-1})^2$$

$$0 = 2x-1$$

$$1 = 2x$$

$$\frac{1}{2} = x$$

$(\frac{1}{2}, 0)$

3. $y = x^4 - 25$

$$y = (0)^4 - 25$$

$$y = -25$$

$(0, -25)$ $(\pm\sqrt{5}, 0)$

$$0 = x^4 - 25$$

$$25 = x^4$$

$$\pm\sqrt[4]{25} = \sqrt[4]{x^4}$$

$$\pm\sqrt[4]{25} = x$$

$$(25)^{1/4} = (5^2)^{1/4}$$

$$= 5^{1/2}$$

$$= \sqrt{5}$$

Examples: Use the algebraic tests to check for symmetry with respect to both axes and the origin.

1. $x - y^2 = 0$

X-axis: $x - (-y)^2 = 0$ ← replace y with $-y$
 $x - y^2 = 0$ ← yields same equation

Y-axis: $-x - y^2 = 0$ ← replace x with $-x$ not the same equation as original

Origin: $-x - (-y)^2 = 0$ ← replace both x & y with opposites
 $-x - y^2 = 0$ ← simplify
 $-(x + y^2) = 0$ ← Not same as original

Conclusion: $x - y^2 = 0$ is sym. w.r.t. x-axis only

2. $y = \frac{1}{x^2 + 1}$

X-axis: $-y = \frac{1}{x^2 + 1}$ is not same as original

Y-axis: $y = \frac{1}{(-x)^2 + 1} = \frac{1}{x^2 + 1}$ same as original

Origin: $-y = \frac{1}{(-x)^2 + 1}$
 $-y = \frac{1}{x^2 + 1}$ → $y = -\frac{1}{x^2 + 1} \Rightarrow$ not same

Conclusion: $y = \frac{1}{x^2 + 1}$ is sym w.r.t. y-axis.

3. $xy = 4$

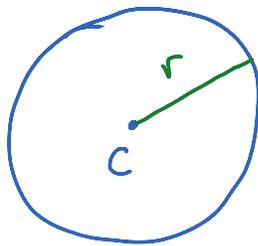
x-axis: $x(-y) = 4$
 $-xy = 4$ NO

y-axis: $(-x)y = 4$
 $-xy = 4$ NO

origin: $(-x)(-y) = 4$
 $xy = 4$ same

Conclusion: The graph of $xy = 4$ is symmetric with respect to the origin.

A circle is defined by the set of all points that are equidistant from a fixed, center, point. Because the definition involves distance, it should be of no surprise that the formula for a circle uses the distance formula.



r = radius, the fixed distance from the center C to all points on the edge of the circle

Standard Form of the Equation of a Circle: The point (x,y) lies on a circle of radius r and center (h,k) if and only if $(x-h)^2 + (y-k)^2 = r^2$. Notice the form $a^2 + b^2 = c^2$

Examples: Write the standard form of the equation of the circle with the given characteristics.

1. center: $(0,0)$; radius: 4
 $h=0, k=0$ $r=4$

$$(x-0)^2 + (y-0)^2 = 4^2$$

simplify: $x^2 + y^2 = 16$

2. Center: (2,-1); radius: 7

$$h=2 \quad r=7$$
$$k=-1$$

$$(x-2)^2 + (y-(-1))^2 = 7^2$$

$$\text{simplify: } (x-2)^2 + (y+1)^2 = 49$$

Examples: Find the center and radius.

1. $x^2 + y^2 = 25$

rewrite in form: $(x-\underline{0})^2 + (y-\underline{0})^2 = \underline{5}^2$

center is (0,0)

radius is 5

2. $(x-1)^2 + (y+3)^2 = 1$

$(x-\underline{1})^2 + (y-\underline{(-3)})^2$

↑ h
↓
↑ k

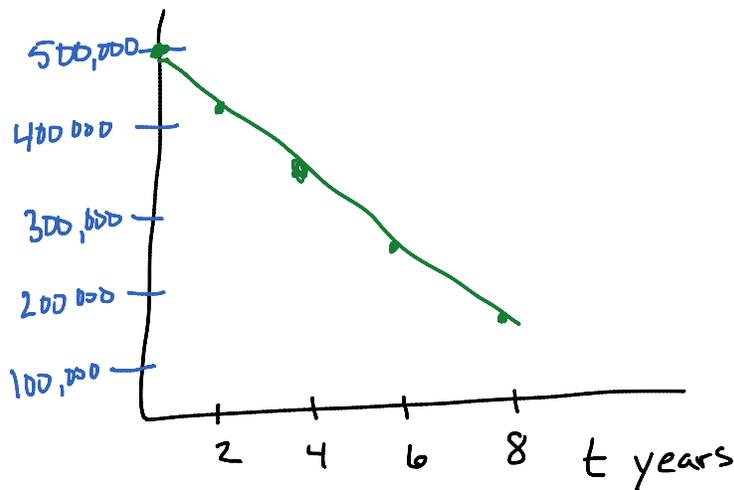
$1=1^2$ so $r=1$

center is (1,-3)

Notice that standard form is $x-h$ and $y-k$ so if we see addition it must be subtracting a negative.

Example: A hospital purchases a new MRI machine for \$500,000. The depreciated value y (reduced value) after t years is given by $y = 500,000 - 40,000t$, $0 \leq t \leq 8$. Sketch a graph of the equation. What do the intercepts mean in the context of the problem?

A value less than zero doesn't make sense so we will graph in the first quadrant only



t	y
2	420,000
4	340,000
6	260,000
8	180,000

y -intercept has $t=0$: $y = 500,000 - 40,000(0)$
 $y = 500,000$

This means the machine cost \$500,000 when new ($t=0$).

t -intercept has $y=0$: $0 = 500,000 - 40,000t$

$$40,000t = 500,000$$

$$t = \frac{500,000}{40,000} = 12.5$$

The machine will be worth \$0 in 12.5 yrs