1.3 Linear Equations in Two Variables

The simplest mathematical model for relating two variables is the linear equation in two variables, $y = mx + b$. The *m* represents the slope, or tilt, of the line and *b* represents the y-intercept of the line. This is referred to as the slope-intercept form of a linear equation.

The slope *m* of a line through (x_1, y_1) and (x_2, y_2) is $m = \frac{y_2 - y_1}{x_2 - x_1}$ where $x_1 \neq x_2$.

Using a generic point (x,y) in place of the second point in the slope formula leads us to the point-slope form of the equation of a line: $y - y_1 = m(x - x_1)$.

$$(x-x_1)m = \frac{y-y_1}{x-x_1}(x-x_1) \longrightarrow (x-x_1)m = y-y_1$$

But it looks better as

$$y-y_1 = m(x-x_1)$$

Examples: Find the slope and y-intercept of the line through the given pair of points. Then write the equation of the line in slope-intercept form.

1. (12,0), (0,-8)

$$m = \frac{-8-0}{0-12} = \frac{-8}{-12} = \frac{-4(2)}{-4(3)} = \frac{2}{3}$$

y-intercept is given as $(0,-8)$

$$y = \frac{2}{3}x - 8$$

2. (2,4), (4,-4)

$$m = \frac{-4-4}{4-2} = \frac{-8}{2} = -4$$

$$y - 4 = -4(x-2)$$

$$y - 4 = -4x + 8$$

equation $\longrightarrow$ $y = -4x + 12$

y-int is $(0,12)$

3. (-2,1), (-4,-5)

$$m = \frac{-5-1}{-4-(-2)} = \frac{-6}{-4+2} = \frac{-6}{-2} = 3$$

$$y-1 = 3(x-(-2))$$
$$y-1 = 3(x+2) \to y-1 = 3x+6$$
$$y = 3x+7 \leftarrow \text{equation}$$

y-int $(0,7)$

Examples: Use the point and slope given to find three other points through which the line passes.

1. (3,-2) with m=-1

$$m = -1 = \frac{-1}{1} = \frac{\text{down } 1}{\text{right } 1}$$

Start $(3,-2)$
$(4,-3)$
$(5,-4)$
$(6,-5)$

Note $m = -1 = \frac{1}{-1} = \frac{\text{up } 1}{\text{Left } 1}$
is a possibility as well

2. (-5,4) with m=2

$$m = 2 = \frac{2}{1} = \frac{\text{up } 2}{\text{right } 1}$$

$(-5,4)$
+1 +2
$(-4,6)$ up 2, right 1
+1 +2
$(-3,8)$
$(-2,10)$

Also possible is

$$m = 2 = \frac{-2}{-1} = \frac{\text{down } 2}{\text{left } 1}$$

Slopes can be used to determine if two nonvertical lines in a plane are parallel or perpendicular.

1. Two distinct nonvertical lines are parallel if and only if their slopes are equal. That is, $m_1 = m_2$

2. Two nonvertical lines are perpendicular if and only if their slopes are negative reciprocals of each other. That is, $m_1 = -\dfrac{1}{m_2}$.

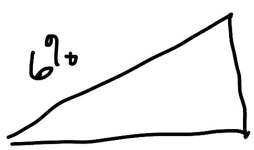
$y = 3x + 6$ and $y = 3x - 17$ are parallel

$y = \frac{1}{3}x - 2$ and $y = -3x + 1$ are perpendicular

$m_1 = \frac{1}{3}$ $\qquad$ $m_2 = -3$

opposite signs, reciprocals

Example: You are driving on a road that has a 6% uphill grade. This means that the slope of the road is 6/100. Approximate the amount of vertical change in your position if you drive 200 feet.



Slope is $\dfrac{\text{Change in } y}{\text{Change in } x} = \dfrac{6}{100}$ $\quad \dfrac{\text{vertical}}{\text{horizontal}}$

$\dfrac{6}{100} = \dfrac{y}{200}$ so $y = 12$ ft

fixed

Example: A microchip manufacturer pays its assembly line workers \$12.25 per hour. In addition, workers receive a piecework rate of \$0.75 per unit produced. Write a linear equation for the hourly wage W in terms of the number of units x produced per hour. → variable pay

$W(x) = 0.75x + 12.25$ per hour

Example: A pharmaceutical salesperson receives a monthly salary of \$2500 plus a commission of 7% of sales. Write a linear equation for the salesperson's monthly wage W in terms of monthly sales S.

variable

fixed

Capital S watch for this on WebAssign

$W(S) = 0.07S + 2500$