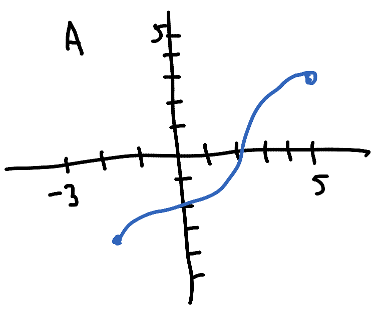
1.5 Analyzing Graphs of Functions

Examples: Find the domain and range of several graphs.

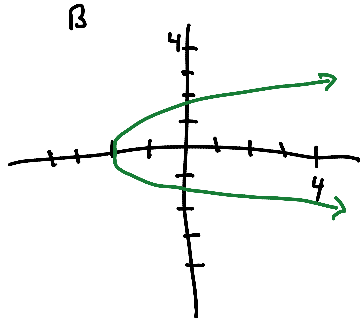
A



Domain $[-2,5)$

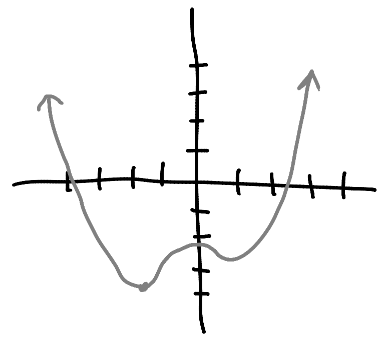
Range $[-3,3)$

B



Domain $[-2,\infty)$ or $x \geq -2$

Range $(-\infty,\infty)$



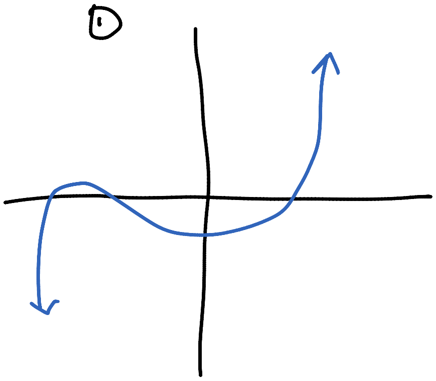
Domain $(-\infty,\infty)$

Range $[-4,\infty)$ approximately

Vertical Line Test for Functions: A set of points in a coordinate plane is the graph of y as a function of x if and only if no vertical line intersects the graph at more than one point.

Examples: Test some graphs for function or not.

1



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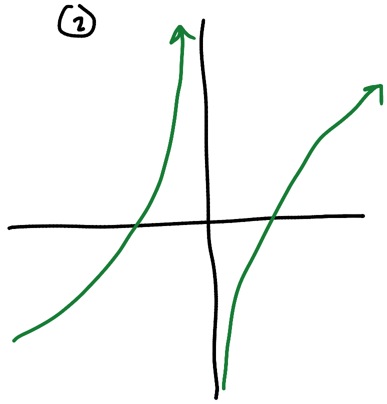

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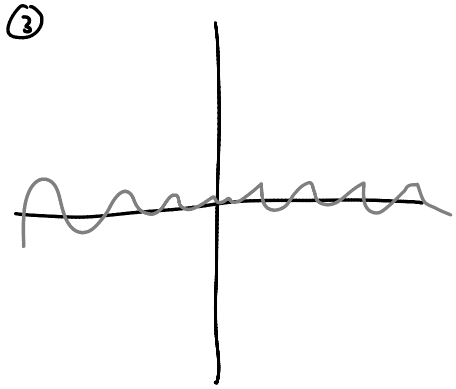
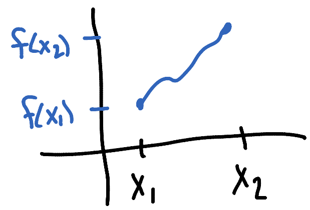


Zeros of a Function: The zeros of a function *f* of x are the x-values for which *f(x) = 0.*

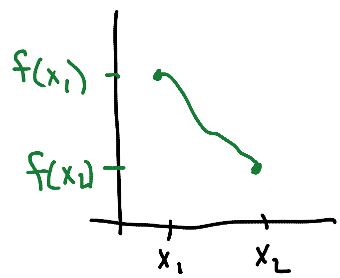
Zeros = Roots = x-intercepts.

Increasing, Decreasing, and Constant Functions:

1. A function is increasing on an interval if, for any x_1 and x_2 in the interval, $x_1 < x_2$ implies $f(x_1) < f(x_2)$.



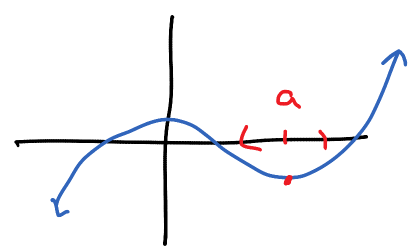
2. A function is decreasing on an interval if, for any x_1 and x_2 in the interval, $x_1 < x_2$ implies $f(x_1) > f(x_2)$.



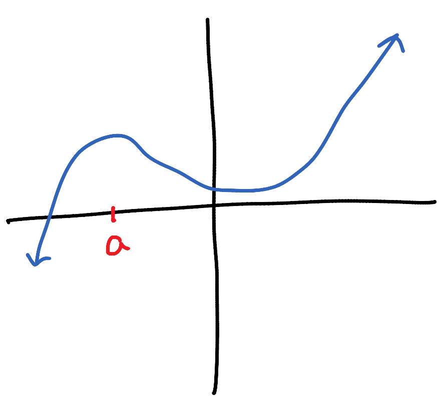
3. A function is constant on an interval if, for any x_1 and x_2 in the interval, $f(x_1) = f(x_2)$.

Relative Minimum: A function value *f(a)*is called a relative minimum of *f* if there exists an interval (x_1, x_2) that contains *a* such that $x_1 < x < x_2$ implies $f(a) \le f(x)$.

Near a, f(a) is the lowest point



Relative Maximum: A function value *f(a)*is called a relative maximum of *f* if there exists an interval (x_1, x_2) that contains *a* such that $x_1 < x < x_2$ implies $f(a) \ge f(x)$.



Near a, f(a) is the highest point.

Note: There may be other higher points, but they are not near enough to $x = a$.

Examples: Find the zeros of the functions algebraically.

Quad formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

1. $f(x) = 3x^2 + 22x - 16$

$0 = 3x^2 + 22x - 16$

$x = \frac{-(22) \pm \sqrt{(22)^2 - 4(3)(-16)}}{2(3)} = \frac{-22 \pm \sqrt{676}}{6} = \frac{-22 \pm 26}{6}$

Simplify: $x = \frac{-22+26}{6} = \frac{2}{3}$ and $x = \frac{-22-26}{6} = -8$

$\boxed{\left(\tfrac{2}{3}, 0\right) \text{ and } (-8, 0)}$

2. $f(x) = \frac{x^2 - 9x + 14}{4x}$

$4x \cdot 0 = \frac{x^2 - 9x + 14}{\cancel{4x}} \cdot \cancel{4x}$

$0 = x^2 - 9x + 14$

$0 = (x-7)(x-2)$

$x - 7 = 0$ or $x - 2 = 0$

$x = 7$ or $x = 2$

$\boxed{(7, 0) \text{ or } (2, 0)}$

3. $f(x) = \sqrt{3x+2}$

$0 = \sqrt{3x+2}$

$(0)^2 = \left(\sqrt{3x+2}\right)^2$

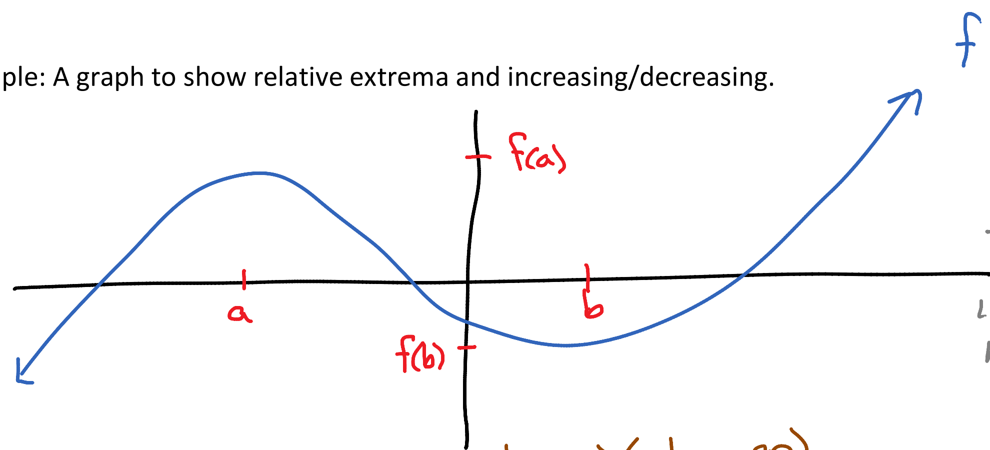
$0 = 3x + 2$

$-2 = 3x$

$-\frac{2}{3} = x$

$\boxed{\left(-\tfrac{2}{3}, 0\right)}$

Example: A graph to show relative extrema and increasing/decreasing.



The point $(a, f(a))$ is a relative maximum

The point $(b, f(b))$ is a relative minimum.

$(-\infty, \quad a)(\quad a, \quad b \quad)(b, \infty)$

The graph increases on $(-\infty, a)$ and (b, ∞). The graph decreases on the interval (a, b).

Average Rate of Change (Avg. r.o.c) – For a nonlinear graph whose slope changes at each point, the average rate of change between any two points is the slope of the line through the two points. This is known as the slope of the secant line.

Example: Find the average rate of change from x_1 to x_2. $\longrightarrow$ $\dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$, the slope formula

1. $f(x) = 3x + 8 \qquad x_1 = 0\, to\, x_2 = 3$

$f(3) = 3(3) + 8 = 9 + 8 = 17$

$f(0) = 3(0) + 8 = 0 + 8 = 8$

$$\text{avg r.o.c} = \frac{f(3) - f(0)}{3 - 0} = \frac{17 - 8}{3} = \frac{9}{3} = 3$$

2. $g(x) = x^2 - 2x + 8 \quad x_1 = 1\, to\, x_2 = 5$

$g(5) = (5)^2 - 2(5) + 8 = 25 - 10 + 8 = 23$

$g(1) = (1)^2 - 2(1) + 8 = 1 - 2 + 8 = 7$

$$\text{avg r.o.c.} = \frac{g(5) - g(1)}{5 - 1} = \frac{23 - 7}{5 - 1} = \frac{16}{4} = 4$$

3. $h(x) = -\sqrt{x+1} + 3 \quad x_1 = 3 \, to \, x_2 = 8$

$h(8) = -\sqrt{8+1} + 3 = -\sqrt{9} + 3 = -3 + 3 = 0$

$h(3) = -\sqrt{3+1} + 3 = -\sqrt{4} + 3 = -2 + 3 = 1$

avg r.o.c. $= \dfrac{h(8) - h(3)}{8-3} = \dfrac{0-1}{8-3} = \dfrac{-1}{5}$