3.4 Exponential and Logarithmic Equations

Strategies for Solving Exponential and Logarithmic Equations

1. Rewrite the original equation in a form that allows the use of the one-to-one properties of exponential and logarithmic functions.

2. Rewrite an exponential equation in logarithmic form and apply the inverse property of logarithmic functions.

3. Rewrite a logarithmic equation in exponential form and apply the inverse property of exponential functions.

Examples: Solve.

1. $4^x = 16$

We can rewrite each side of this equation with the same base in order to use the one-to-one property of equality:

$4^x = 4^2$ therefore $x = 2$

2. $\left(\frac{1}{4}\right)^x = 64$

rewriting each side we get $\frac{1}{4} = 4^{-1}$ and $64 = 4^3$

which becomes $(4^{-1})^x = 4^3$ or $4^{-x} = 4^3$. Therefore $-x = 3$ and $x = -3$.

3. $\ln x - \ln 5 = 0$

The easiest way would be to add $\ln 5$ to both sides for the use of the one-to-one property of equality:

$\ln x = \ln 5$ becomes $x = 5$.

4. $e^x = 5$

There is no chance of rewriting to the same base so we convert forms: If $e^x = 5$, then $x = \ln 5 \approx 1.609$

5. $\log x = -2$

Switch forms with a base of 10

$10^{-2} = x$

so $\frac{1}{10^2} = x$ or $\frac{1}{100} = x$

6. $\log_5 x = \frac{1}{2}$

$5^{1/2} = x$

$\sqrt{5} = x \approx 2.236$

7. $e^{2x} = e^{x^2-8}$

Same base so: $2x = x^2 - 8$

to solve a quadratic, set it equal to zero and factor or use the formula

$0 = x^2 - 2x - 8$

$0 = (x-4)(x+2)$

$x = 4$ or $x = -2$

8. $2(5^x) = 32$

Isolate the exponential: $\frac{2(5^x)}{2} = \frac{32}{2}$ is $5^x = 16$

now rewrite to solve: $x = \log_5(16) \approx 1.723$

x is the power to which I raise 5 to get 16

9. $6^x + 10 = 47$

Isolate: $6^x + 10 = 47$

$\quad -10 \quad -10$

$6^x = 37$

rewrite: $x = \log_6(37) \approx 2.015$

10. $4^{-3t} = 0.10$

$\frac{-3t}{-3} = \frac{\log_4(0.10)}{-3}$

$t = \frac{\log_4(0.10)}{-3} = \frac{\log(0.10)/\log(4)}{-3} \approx 0.554$

11. $2^{x-3}=32$

let it be easy when you can

$2^{x-3}=2^5$

so $x-3=5$

$x=8$

12. $8\left(3^{6-x}\right)=40$

$\frac{8\left(3^{6-x}\right)}{8}=\frac{40}{8}$

$3^{6-x}=5$

$6-x=\log_3(5)$

$-6 \qquad -6$

$-x=\log_3(5)-6$

$\frac{-x}{-1}=\frac{\log_3(5)-6}{-1}$

$x=6-\log_3(5)$

$x\approx 4.535$

13. $e^{2x}-5e^x+6=0$

this is called quadratic in form because it looks like $y^2-5y+6=0$

for this reason we factor: $(e^x-3)(e^x-2)=0$

$e^x-3=0$ or $e^x-2=0$

$e^x=3$ or $e^x=2$

$x=\ln 3\approx 1.099$, $x=\ln 2\approx 0.693$

14. $e^{2x}+9e^x-36=0$

$(e^x+12)(e^x-3)=0$

$e^x+12=0$ or $e^x-3=0$

$e^x\neq -12$ or $e^x=3$

not in range of e^x $\qquad$ $x=\ln 3\approx 1.099$

15. $\ln(x+1)-\ln(x-2)=\ln x$

Rewrite the left: $\ln\frac{x+1}{x-2}=\ln x$

Use 1-1 property: $\frac{x+1}{x-2}=x$

Solve: $\cancel{(x-2)}\cdot\frac{x+1}{\cancel{x-2}}=x\cdot(x-2)$

$x+1=x^2-2x$

$-x\ -1 \qquad -x\ -1$

$0=x^2-3x-1$

$\boxed{x=\frac{3+\sqrt{13}}{2}}$ or $x\neq\frac{3-\sqrt{13}}{2}$

bigger than 3 so ok

negative so not ok

16. $\log_4 x - \log_4(x-1) = \frac{1}{2}$

Combine: $\log_4 \frac{x}{x-1} = \frac{1}{2}$

Rewrite: $4^{1/2} = \frac{x}{x-1}$

simplify: $\sqrt{4} = 2 = \frac{x}{x-1}$

solve: $(x-1)2 = \frac{x}{\cancel{x-1}} \cdot \cancel{(x-1)}$

$2x-2 = x$

$x = 2$

17. $\log_3 x + \log_3(x-8) = 2$

$\log_3(x^2-8x) = 2$

$3^2 = x^2 - 8x$

$9 = x^2 - 8x$

$0 = x^2 - 8x - 9$

$0 = (x-9)(x+1)$

$x-9=0$ or $x+1=0$

$x=9$ or $x \neq -1$

not in the domain of the original expressions so not an acceptable answer