

4.2 Trigonometric Functions: The Unit Circle

Back in chapter one, we discussed the circles. The unit circle is given by $x^2 + y^2 = 1$ because the radius is one. We can define all six trig functions using this unit circle. The trig functions are sine, cosine, tangent, cotangent, secant, and cosecant.

Definitions of Trigonometric Functions: Let t be a real number and let (x, y) be a point on the unit circle corresponding to t . (We picture the real number line wrapped around the unit circle.)

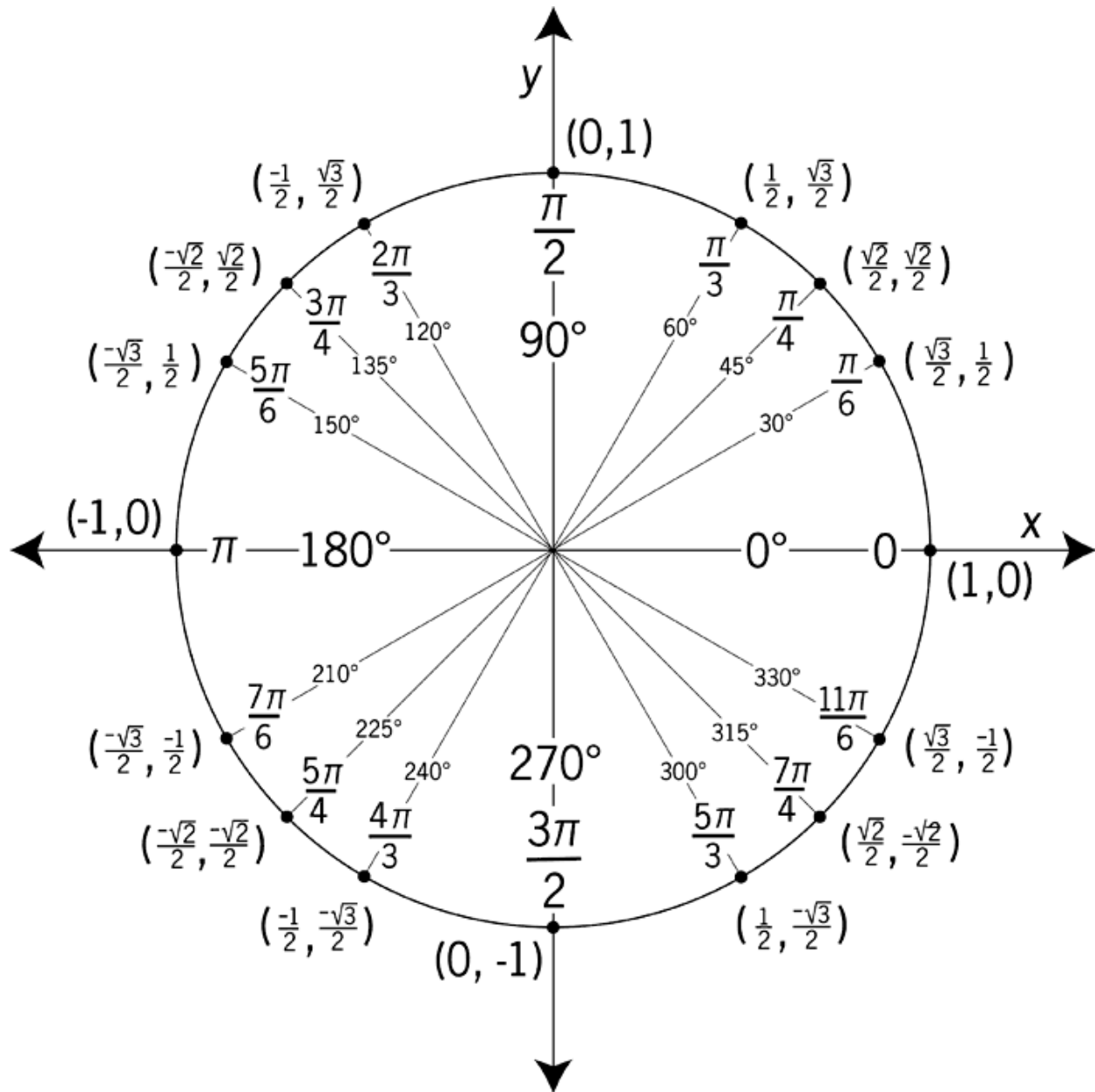
$$\sin t = y \qquad \cos t = x \qquad \tan t = \frac{y}{x}, x \neq 0$$

$$\csc t = \frac{1}{y}, y \neq 0 \qquad \sec t = \frac{1}{x}, x \neq 0 \qquad \cot t = \frac{x}{y}, y \neq 0$$

In section 1 of chapter 4 we looked at converting between radians and degrees. Sometimes, having things memorized will be in your best interest. I encourage you to memorize the following table. I'll explain why in class.

Degrees	Radians	$\sin t$	$\cos t$	$\tan t$
0	0	$\frac{\sqrt{0}}{2} = 0$	$\frac{\sqrt{4}}{2} = 1$	0
30	$\pi/6$	$\frac{\sqrt{1}}{2} = \frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
45	$\pi/4$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}/2}{\sqrt{2}/2} = 1$
60	$\pi/3$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{1}}{2} = \frac{1}{2}$	$\frac{\sqrt{3}/2}{1/2} = \sqrt{3}$
90	$\pi/2$	$\frac{\sqrt{4}}{2} = \frac{2}{2} = 1$	$\frac{\sqrt{0}}{2} = 0$	undefined

We can divide the unit circle up into several pieces and find the values of the six functions. It would be in your best interest to memorize this information. You will need it in calculus and every other higher math class you take from this day forward.



$$\sin t = y$$

$$\cos t = x$$

$$\tan t = \frac{y}{x}, x \neq 0$$

$$\csc t = \frac{1}{y}, y \neq 0$$

$$\sec t = \frac{1}{x}, x \neq 0$$

$$\cot t = \frac{x}{y}, y \neq 0$$

Examples: Find the point (x,y) on the unit circle that corresponds to the real number t.

1. $t = \pi$

$(-1, 0)$

2. $t = \pi/3$

$(\frac{1}{2}, \frac{\sqrt{3}}{2})$

3. $t = 5\pi/3$

$(\frac{1}{2}, -\frac{\sqrt{3}}{2})$

Examples: Evaluate, if possible, the sine, cosine, and tangent of the real number.

1. $t = \pi/4$ point $(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$ $\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$ $\cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$\tan \frac{\pi}{4} = \frac{\sqrt{2}/2}{\sqrt{2}/2} = 1$

Positive Coterminal

2. $t = -\pi/6 = \frac{11\pi}{6}$ point $(\frac{\sqrt{3}}{2}, -\frac{1}{2})$

$\sin(-\frac{\pi}{6}) = -\frac{1}{2}$ $\cos(-\frac{\pi}{6}) = \frac{\sqrt{3}}{2}$ $\tan(-\frac{\pi}{6}) = \frac{-1/2}{\sqrt{3}/2} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

3. $t = -7\pi/4 = \frac{\pi}{4}$ $(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$

$\sin(-\frac{7\pi}{4}) = \frac{\sqrt{2}}{2}$ $\cos(-\frac{7\pi}{4}) = \frac{\sqrt{2}}{2}$ $\tan(-\frac{7\pi}{4}) = 1$

4. $t = 2\pi/3$ $(-\frac{1}{2}, \frac{\sqrt{3}}{2})$

$\sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2}$ $\cos \frac{2\pi}{3} = -\frac{1}{2}$ $\tan \frac{2\pi}{3} = \frac{\sqrt{3}/2}{-1/2} = -\sqrt{3}$

Examples: Evaluate, if possible, the six trig functions of the real number.

1. $t = 5\pi/6$ point $(-\frac{\sqrt{3}}{2}, \frac{1}{2})$

$\sin t = \frac{1}{2}$

$\csc t = 2$

$\cos t = -\frac{\sqrt{3}}{2}$

$\sec t = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$

$\tan t = \frac{1}{-\sqrt{3}} = -\frac{\sqrt{3}}{3}$

$\cot t = -\sqrt{3}$

← reciprocals of the functions above

$$2. \quad t = 3\pi/4 \quad \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$$

$$\sin t = \frac{\sqrt{2}}{2} \quad \cos t = -\frac{\sqrt{2}}{2} \quad \tan t = -1$$

$$\csc t = \frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2} \quad \sec t = \frac{-2}{\sqrt{2}} = -\sqrt{2} \quad \cot t = -1$$

$$3. \quad t = -\pi/2 \quad (0, -1)$$

$$\sin t = -1 \quad \cos t = 0 \quad \tan t = \frac{-1}{0} = \text{undefined}$$

$$\csc t = -1 \quad \sec t = \frac{1}{0} = \text{undefined} \quad \cot t = \frac{0}{-1} = 0$$

Domain of Periodic Function: A function f is periodic if there exists a positive real number c such that $f(t+c) = f(t)$ for all t in the domain of f . The smallest number c for which f is periodic is called the period of f .

Fact: Both sine and cosine have a period of 2π .

Even and Odd Trig Functions:

The cosine and secant functions are even so, $\cos(-t) = \cos t$ and $\sec(-t) = \sec t$.

The other four trig functions are odd so, $\sin(-t) = -\sin t$, $\tan(-t) = -\tan t$,

$\csc(-t) = -\csc t$, and $\cot(-t) = -\cot t$.