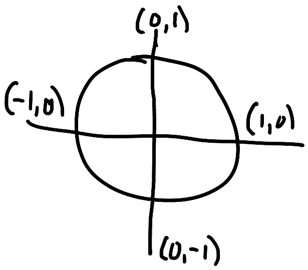
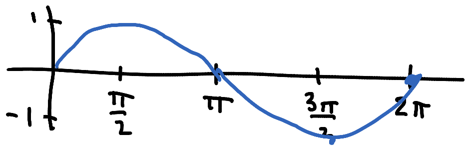
4.5 Graphs of Sine and Cosine Functions

To graph the sine or cosine function by hand, you need to know the five key points for one period $[0, 2\pi]$. We can find these points by examining the unit circle and "unraveling" it into a linear graph.

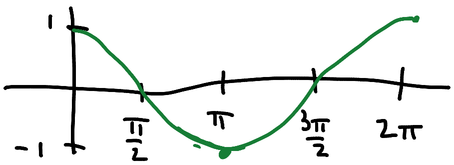
Example: The development of $y = \sin x$ and $y = \cos x$.



Sine is the y-values from the unit circle
(0rad, 0) $(\pi/2, 1)$ $(\pi, 0)$ $(\frac{3\pi}{2}, -1)$ and $(2\pi, 0)$



Cosine is the x-values from unit circle: $(0,1)$, $(\pi/2, 0)$, $(\pi, -1)$, $(\frac{3\pi}{2}, 0)$ and $(2\pi, 1)$



there will be a quiz over these two graphs!

Definition of Amplitude of Sine and Cosine Curves – The amplitude of $y = a\sin x$ and $y = a\cos x$ represents half the distance between the maximum and minimum values of the function and is given by Amplitude = $|a|$.

so always positive

notice this is just a vertical stretch/shrink from chapter 1

Period of Sine and Cosine Functions – Let *b* be a positive real number. The period of $y = a\sin bx$ and $y = a\cos bx$ is given by Period = $\frac{2\pi}{b}$.

this is a horizontal shrink/stretch from Chapter 1

Examples: Find the period and amplitude.

1. $y = -4\sin x$

$a = -4$
$b = 1$

$Amp = |-4| = 4$

$Period = \frac{2\pi}{1} = 2\pi$

2. $y = -\cos\frac{2x}{3}$

$a = -1$
$b = 2/3$

$Amp = |-1| = 1$

$Period = \frac{2\pi}{\frac{2}{3}} = 2\pi \cdot \frac{3}{2} = 3\pi$

3. $y = \frac{1}{5}\sin 6x$

$a = \frac{1}{5}$ $b = 6$

$Amp = |1/5| = \frac{1}{5}$

$Period = \frac{2\pi}{6} = \frac{\pi}{3}$

4. $y = \frac{1}{4}\sin 2\pi x$

$a = \frac{1}{4}$ $b = 2\pi$

$Amp = |\frac{1}{4}| = \frac{1}{4}$

$Period = \frac{2\pi}{2\pi} = 1$

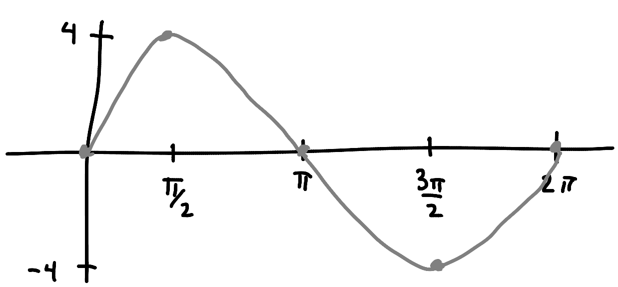
Graphs of Sine and Cosine Functions – The graphs of $y = a\sin(bx - c)$ and $y = a\cos(bx - c)$ have the following characteristics. (Assume *b* > *0*.)

a. Amplitude = $|a|$
b. Period = $\frac{2\pi}{b}$.
c. The left endpoint of a one-cycle interval can be determined by solving the equation $bx - c = 0$.
d. The right endpoint of a one-cycle interval can be determined by solving the equation $bx - c = 2\pi$.
e. The number *c/b* is the phase shift.

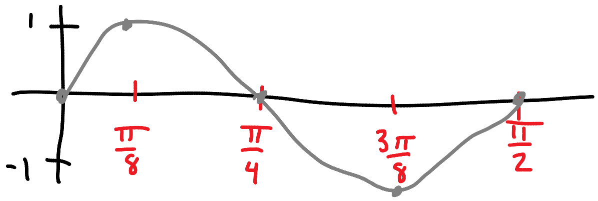
Examples: Sketch the graph of the function.

1. $y = 4\sin x$

$Amp = 4$ $Period = \frac{2\pi}{1} = 2\pi$ the height is the only change



2. $y = \sin 4x$ Amp = 1 Period = $\frac{2\pi}{4} = \frac{\pi}{2}$ (horizontal shrink by a factor of 4)



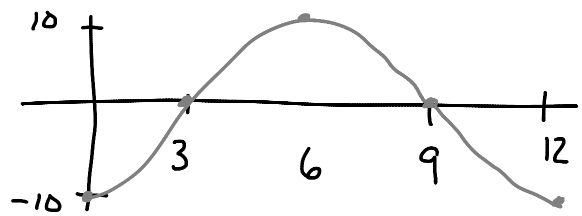
Same basic graph, all that changes are the scaling of the axes.

3. $y = -10\cos\frac{\pi x}{6}$

vert reflection

Amp = $|-10| = 10$

Period = $\frac{2\pi}{\frac{\pi}{6}} = 2\pi \cdot \frac{6}{\pi} = 12$



remember, this is taken from a circle so it should have curves, not lines.

4. $y = 4\cos\left(x + \frac{\pi}{4}\right) + 2$

Amp = 4

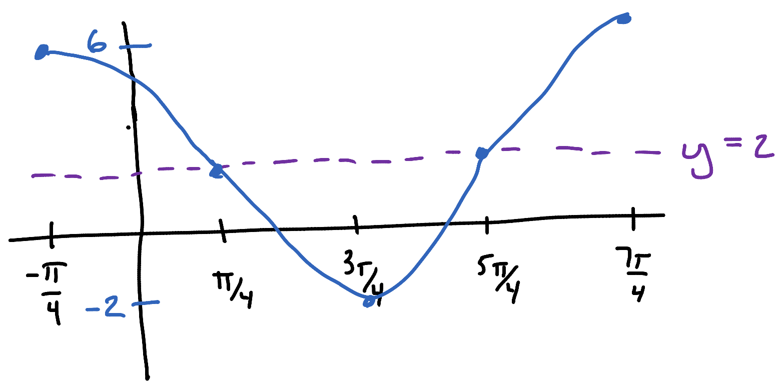
Period = 2π

shifted up 2

phase shift $x + \frac{\pi}{4} = 0$

$x = -\frac{\pi}{4}$

Start at $-\pi/4$



to find x-values for graph we solve mini-equations.

equal to regular x-value of normal cosine graph.

$x + \frac{\pi}{4} = 0$	$x + \frac{\pi}{4} = \frac{\pi}{2}$	$x + \frac{\pi}{4} = \pi$	$x + \frac{\pi}{4} = \frac{3\pi}{2}$	$x + \frac{\pi}{4} = 2\pi$
$x = -\frac{\pi}{4}$	$x = \frac{\pi}{4}$	$x = \frac{3\pi}{4}$	$x = \frac{5\pi}{4}$	$x = \frac{7\pi}{4}$

to find y-values, go up or down 4 from midline of $y = 2$.

Examples: Describe the sequence of transformations from the parent functions.

1. $g(x) = \sin(2x + \pi)$

horizontal shrink by a factor of 2: period $= \frac{2\pi}{2} = \pi$

phase shift: $2x + \pi = 0$

$2x = -\pi$

$x = -\pi/2$

(left $\pi/2$)

2. $g(x) = 1 + \cos(x + \pi)$

vertical shift up 1

left π

3. $g(x) = 4 - \sin(3x - \pi)$

up 4

reflected across y-axis

horizontal shrink by 3

period $= \frac{2\pi}{3}$

phase shift $3x - \pi = 0$

$3x = \pi$

$x = \pi/3$

(right $\pi/3$)

Examples: When tuning a piano, a technician strikes a tuning fork for the A above middle C and sets up a wave motion that can be approximated by $y = 0.001\sin \underline{880}\pi t$, where *t* is the time (in seconds).

(a) What is the period of the function?

Period $= \frac{2\pi}{880\pi} = \frac{1}{440}$

(b) The frequency *f* is given by $f = 1/p$. What is the frequency of the note?

$f = \frac{1}{p} = \frac{1}{\frac{1}{440}} = 1 \cdot \frac{440}{1} = 440$ Hz

Examples: A Ferris wheel is built such that the height *h* (in feet) above ground of a seat on the wheel at time *t* (in seconds) can be modeled by $h(t) = 53 + 50\sin\left(\frac{\pi}{10}t - \frac{\pi}{2}\right)$.

(a) Find the period of the model. What does the period tell you about the ride?

period $= \frac{2\pi}{b}$ where $b = \frac{\pi}{10}$

$p = \frac{2\pi}{\frac{\pi}{10}} = 2\pi \cdot \frac{10}{\pi} = 20$

It takes 20 seconds to go once around this ride.

(b) Find the amplitude of the model. What does the amplitude tell you about the ride?

Amp $= |50| = 50$ ft.

From the centerline of 53 ft, this ride reaches a max height of 103 ft and a minimum height of 3 ft.