4.6 Graphs of Other Trigonometric Functions

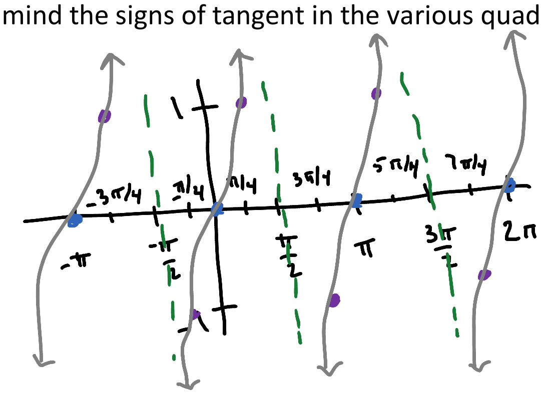
To find the graph of $y = \tan x$ we need to keep in mind that $\tan x = \dfrac{\sin x}{\cos x}$.

We know that tanx will be zero when sinx is zero

We know that tanx will be undefined when cosx is zero asymptotes

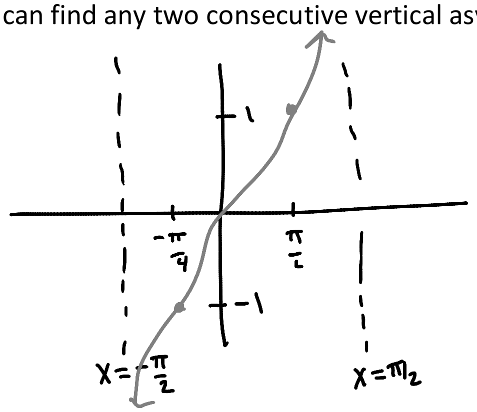
Also, tanx = 1 when sinx and cosx are equal (π/4).

Keeping in mind the signs of tangent in the various quadrants we can plot the graph.



notice how the graph repeats itself every π units!

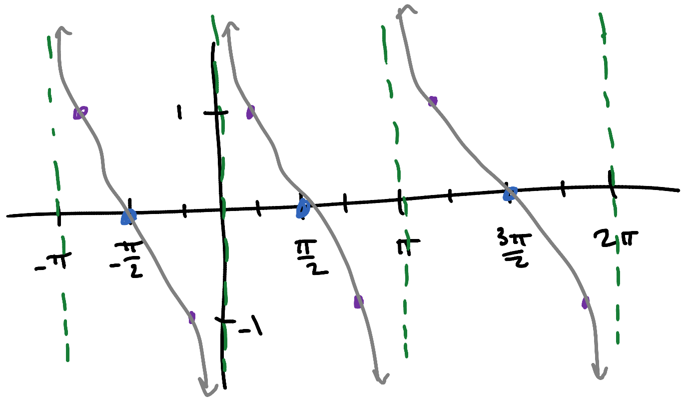
Since the vertical asymptotes of tanx are when cosx = 0, the vertical asymptotes are the lines $x = -\dfrac{\pi}{2}$ and $x = \dfrac{\pi}{2}$. For various shifts we can find any two consecutive vertical asymptotes by solving $bx - c = -\dfrac{\pi}{2}$ and $bx - c = \dfrac{\pi}{2}$.



one period, the standard period of y = tanx.

We can use similar logic to discover the cotangent graph as well.

$y = \cot x = \dfrac{\cos x}{\sin x}$



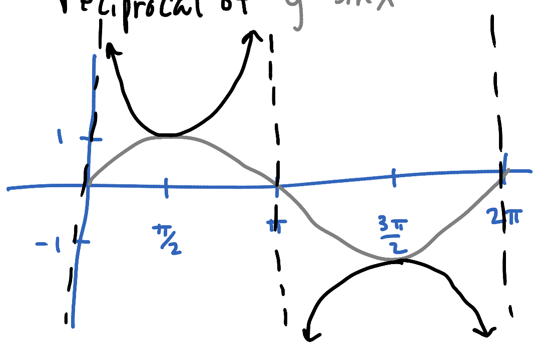
Zeros of cos x are zeros of cot x

Zeros of sin x are asymptotes of cot x

cot x = 1 when sin x = cos x (π/4)
take into consideration quadrants

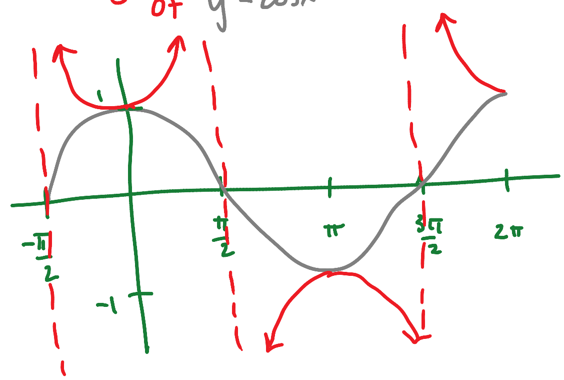
To find the graphs of the reciprocal functions secant and cosecant, all we have to do is start with the graphs of sine and cosine.

y = csc x is the reciprocal of y = sin x



Ignore the y = sin x graph as it is only a pattern to get the other.

y = sec x is the reciprocal of y = cos x

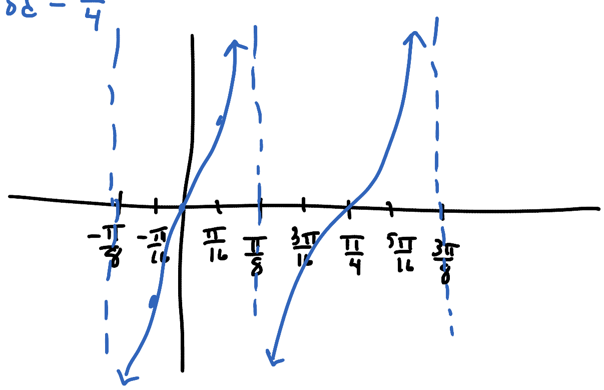


Ignore the y = cos x graph as it is only a pattern for y = sec x

Examples: Sketch the graph of the function. Include two full periods for tangent and cotangent.

1. $y = \tan 4x$

period = $\frac{\pi}{4}$



$4x = -\frac{\pi}{2}$ $\quad x = -\frac{\pi}{8}$

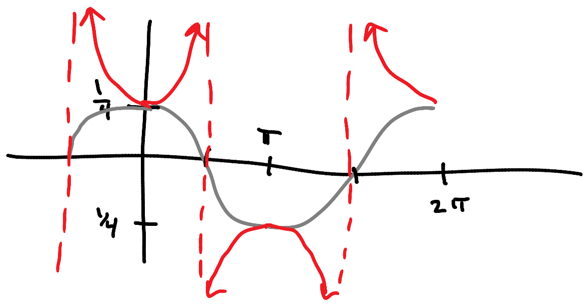
$4x = -\frac{\pi}{4}$ $\quad x = -\frac{\pi}{16}$

$4x = 0$ $\quad x = 0$

$4x = \frac{\pi}{4}$ $\quad x = \frac{\pi}{16}$

$4x = \frac{\pi}{2}$ $\quad x = \frac{\pi}{8}$

2. $y = \frac{1}{4}\sec x$ If this were $y = \frac{1}{4}\cos x$, the amplitude would be $\frac{1}{4}$.
Secant doesn't have an amplitude but the $\frac{1}{4}$ is still important.

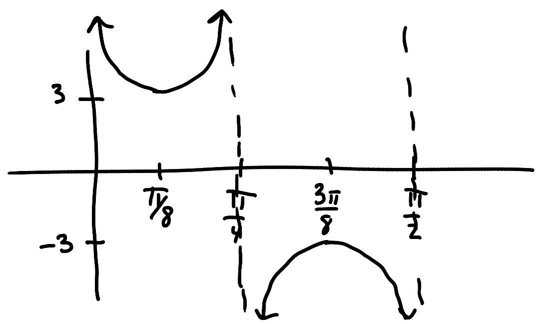


the graph in red only.
Use $y = \cos x$ as necessary, but try to get to a point that you don't need it.

3. $y = 3\csc 4x$

max/min values

horizontal shrink $per = \frac{2\pi}{4} = \frac{\pi}{2}$



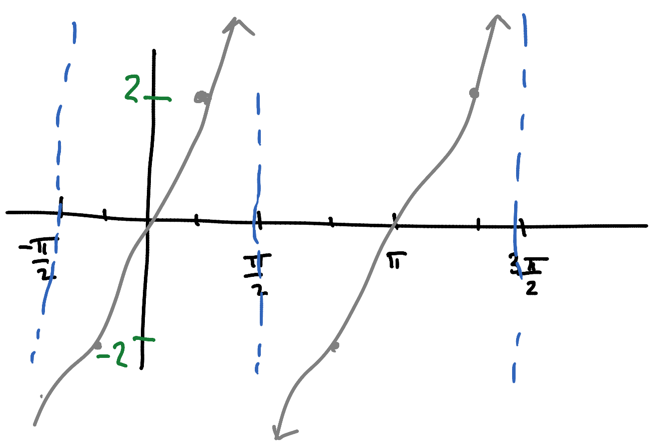
you can draw in $y = 3\sin 4x$ if necessary

4. $y = 2\cot\left(x + \frac{\pi}{2}\right)$

$x + \frac{\pi}{2} = 0$ (where cotangent starts)

$x = -\frac{\pi}{2}$ is phase shift

like secant and cosecant, tangent and cotangent do not have an amplitude, but the 2 matters in scaling the y-axis



notice that the graph of cotangent shifted left $\frac{\pi}{2}$ looks exactly like the graph of the tangent.
This is what it means to be a co-function.