4.7 Inverse Trigonometric Functions

Definition of Inverse Sine Function – The inverse sine function is defined by $y = \arcsin x$ if and only if $\sin y = x$ where $-1 \le x \le 1$ and $-\pi/2 \le y \le \pi/2$. The domain of $y = \arcsin x$ is [-1,1] and the range is [-π/2,π/2].

To graph the inverse sine function, use the process of inverses by switching x and y and then graph. It is important to keep in mind the domain and range. These graphs end and do not continue forever.

The full development of the Big Three inverse trigonometric functions is given in the PowerPoint.

Definitions of the Inverse Trig Functions –

Function	Domain	Range
$y = \arcsin x$ *iff* $\sin y = x$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
$y = \arccos x$ *iff* $\cos y = x$	$-1 \le x \le 1$	$0 \le y \le \pi$
$y = \arctan x$ *iff* $\tan y = x$	$-\infty < x < \infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$

Examples: Evaluate the expression without using a calculator.

1. $\arcsin 0$

For what angle is the sine function equal to zero In the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$? $\arcsin 0 = 0$

2. $\arccos 0$

When is cosine zero in $[0, \pi]$? $\arccos 0 = \pi/2$

3. $\arctan(1)$

When is tangent equal to 1 in $(-\frac{\pi}{2}, \frac{\pi}{2})$?

$\arctan(1) = \frac{\pi}{4}$

4. $\arctan \sqrt{3}$

When is tangent equal to $\sqrt{3}$? This occurs when sine is $\frac{\sqrt{3}}{2}$ and cosine is $\frac{1}{2}$ which occurs at $\frac{\pi}{3}$.

$$\arctan \sqrt{3} = \frac{\pi}{3}$$

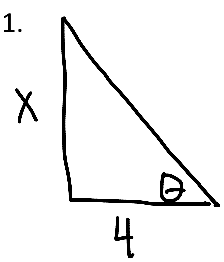
5. $\cos^{-1} 1$

When is cosine equal to 1?

$$\cos^{-1}(1) = \arccos(1) = 0$$

Examples: Use an inverse trigonometric function to write θ as a function of x.

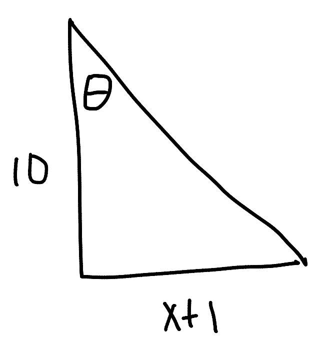
1.



we know $\tan\theta = \frac{x}{4}$ so now solve for θ:

$$\theta = \arctan\left(\frac{x}{4}\right)$$

2.



we know $\tan\theta = \frac{x+1}{10}$. Solving for θ we get:

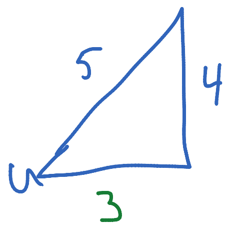
$$\theta = \arctan\left(\frac{x+1}{10}\right)$$

Examples: Find the exact value of the expression. (Hint: Sketch a right triangle.)

1. $\sec\left(\arcsin\frac{4}{5}\right)$ $= \sec u = \boxed{\frac{5}{3}}$ using the filled in triangle.

Let $u = \arcsin\left(\frac{4}{5}\right)$

Then $\sin u = \frac{4}{5}$



$5^2 = 4^2 + x^2$

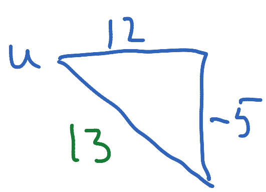
$25 - 16 = x^2$

$9 = x^2$

2. $\csc\left[\arctan\left(-\frac{5}{12}\right)\right]$ $\longrightarrow$ $\boxed{\csc u = \frac{13}{-5}}$

Let $u = \arctan\left(-\frac{5}{12}\right)$ * quad 4

Then $\tan u = \frac{-5}{12}$



$12^2 + (-5)^2 = r^2$

$144 + 25 = r^2$

$169 = r^2$

3. $\sec\left[\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)\right]$

Without finding the triangle, I know the inverse sine is negative in q4 and $\frac{\sqrt{2}}{2}$ with reference angle $\frac{\pi}{4}$. In q4 secant is positive and is the reciprocal of $\cos\left(-\frac{\pi}{4}\right)$. $\sec\left[\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)\right] = \sqrt{2}$