Chapter Five: Analytic Trigonometry

5.1 Using Fundamental Identities

The Fundamental Trigonometric Identities include:

Reciprocal Identities

$$\sin u = \frac{1}{\csc u} \qquad \cos u = \frac{1}{\sec u} \qquad \tan u = \frac{1}{\cot u}$$

$$\csc u = \frac{1}{\sin u} \qquad \sec u = \frac{1}{\cos u} \qquad \cot u = \frac{1}{\tan u}$$

Quotient Identities

$$\tan u = \frac{\sin u}{\cos u} \qquad \cot u = \frac{\cos u}{\sin u}$$

Pythagorean Identities

$$\sin^2 u + \cos^2 u = 1 \qquad 1 + \tan^2 u = \sec^2 u \qquad 1 + \cot^2 u = \csc^2 u$$

Co-function Identities

$$\sin\left(\frac{\pi}{2} - u\right) = \cos u \qquad \cos\left(\frac{\pi}{2} - u\right) = \sin u \qquad \tan\left(\frac{\pi}{2} - u\right) = \cot u$$

$$\csc\left(\frac{\pi}{2} - u\right) = \sec u \qquad \sec\left(\frac{\pi}{2} - u\right) = \csc u \qquad \cot\left(\frac{\pi}{2} - u\right) = \tan u$$

Even/Odd Identities

$$\sin(-u) = -\sin u \qquad \cos(-u) = \cos u \qquad \tan(-u) = -\tan u$$

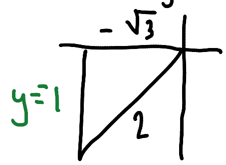
$$\csc(-u) = -\csc u \qquad \sec(-u) = \sec u \qquad \cot(-u) = -\cot u$$

Using only these identities we can do a lot with trig functions.

Examples: Use the given values to evaluate (if possible) all six trig functions.

1. $\tan x = \dfrac{\sqrt{3}}{3}, \ \cos x = -\dfrac{\sqrt{3}}{2}$

tangent is positive and cosine is negative in quadrant 3:



$(-\sqrt{3})^2 + y^2 = 2^2$

$3 + y^2 = 4$

$y^2 = 1$

$\sin x = -\dfrac{1}{2}$ $\qquad \csc x = -2$

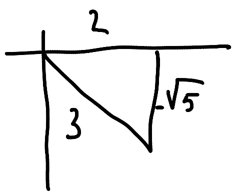
$\cos x = -\dfrac{\sqrt{3}}{2}$ $\qquad \sec x = -\dfrac{2}{\sqrt{3}} = \dfrac{-2\sqrt{3}}{3}$

$\tan x = \dfrac{\sqrt{3}}{3}$ $\qquad \cot x = \dfrac{\sqrt{3}}{1} = \sqrt{3}$

2. $\sec\phi = \dfrac{3}{2}, \ \csc\phi = -\dfrac{3\sqrt{5}}{5}$

positive x $\qquad$ negative y

$\boxed{Q4}$



$\sin\phi = \dfrac{-5}{3\sqrt{5}} = -\dfrac{\sqrt{5}}{3} \leftarrow \csc\phi = -\dfrac{3\sqrt{5}}{5}$

$\cos\phi = \dfrac{2}{3} \leftarrow \sec\phi = \dfrac{3}{2}$

$\tan\phi = -\dfrac{\sqrt{5}}{2}$ $\qquad \cot\phi = \dfrac{2}{-\sqrt{5}} = \dfrac{-2\sqrt{5}}{5}$

Ex: Simplify the trigonometric expression.

1. $\tan x \csc x$

$LHS = \tan x \cdot \dfrac{1}{\sin x} = \dfrac{\sin x}{\cos x} \cdot \dfrac{1}{\sin x} = \dfrac{1}{\cos x} = \sec x$

Notice that we are not given a clear goal with a RHS. Just simplify as much as possible.

2. $(1 - \cos^2 x)(\csc x)$

$LHS = \sin^2 x \csc x = \sin^2 x \cdot \dfrac{1}{\sin x} = \dfrac{\sin^2 x}{\sin x} = \sin x$

3. $\sec^4 x - \tan^4 x$

$$LHS = (\sec^2 x)^2 - (\tan^2 x)^2 = (\sec^2 x - \tan^2 x)(\sec^2 x + \tan^2 x)$$

difference of squares formula

$$= (1 + \tan^2 x - \tan^2 x)(1 + \tan^2 x + \tan^2 x)$$

$$= (1)(1 + 2\tan^2 x)$$

$$= 1 + 2\tan^2 x$$

4. $\sin x \cot(-x)$

$$LHS = \sin x(-\cot x) = \sin x\left(\frac{-\cos x}{\sin x}\right) = -\cos x$$

5. $\dfrac{\csc\theta}{\sec\theta}$

$$LHS = \frac{\frac{1}{\sin\theta}}{\frac{1}{\cos\theta}} = \frac{1}{\sin\theta} \cdot \frac{\cos\theta}{1} = \frac{\cos\theta}{\sin\theta} = \cot\theta$$

6. $\sin\beta \tan\beta + \cos\beta$

$$LHS = \sin\beta \cdot \frac{\sin\beta}{\cos\beta} + \cos\beta = \frac{\sin^2\beta}{\cos\beta} + \frac{\cos\beta}{1} \cdot \frac{\cos\beta}{\cos\beta}$$

$$= \frac{\sin^2\beta}{\cos\beta} + \frac{\cos^2\beta}{\cos\beta}$$

$$= \frac{\sin^2\beta + \cos^2\beta}{\cos\beta}$$

$$= \frac{1}{\cos\beta}$$

$$= \sec\beta$$