5.2 Verifying Trigonometric Identities

Guidelines for Verifying Trigonometric Identities –

1. Work with one side of the equation at a time. It is often better to work with the more complicated side first.

2. Look for opportunities to factor an expression, add fractions, square a binomial, or create a monomial denominator.

3. Look for opportunities to use the fundamental identities. Note which functions are in the final expression you want. Sines and cosines pair up well, as do secants and tangents and cosecants and cotangents.

4. If the preceding guidelines do not help, try converting all terms to sines and cosines.

5. Always try something! Even paths that lead to dead ends provide insights.

Examples: Verify the identities.

1. $\sec y \cos y = 1$

$LHS = \frac{1}{\cos y} \cdot \cos y = \frac{\cos y}{\cos y} = 1$ ✓

2. $\cos^2 \beta - \sin^2 \beta = 2\cos^2 \beta - 1$

$LHS = \cos^2\beta - (1 - \cos^2\beta) = \cos^2\beta - 1 + \cos^2\beta = 2\cos^2\beta - 1$

3. $\frac{\cot^3 y}{\csc y} = \cos y\left(\csc^2 y - 1\right)$

LHS = ...? I don't even know where to begin.

$RHS = \cos y(\cot^2 y) = \cos y\left(\frac{\cos^2 y}{\sin^2 y}\right) = \frac{\cos^3 y}{\sin^2 y}$ ... now what? Go back to the left and try something

$LHS = \frac{\frac{\cos^3 y}{\sin^3 y}}{\frac{1}{\sin y}} = \frac{\cos^3 y}{\sin^3 y} \cdot \frac{\sin y}{1} = \frac{\cos^3 y}{\sin^2 y}$

We rewrote both sides to the same thing. The identity is verified.

4. $\dfrac{\sec\theta - 1}{1 - \cos\theta} = \sec\theta$

$$LHS = \frac{\frac{1}{\cos\theta} - 1}{1 - \cos\theta} = \frac{\frac{1}{\cos\theta} - \frac{\cos\theta}{\cos\theta}}{1 - \cos\theta} = \frac{\frac{1 - \cos\theta}{\cos\theta}}{1 - \cos\theta} = \frac{1 - \cos\theta}{\cos\theta} \cdot \frac{1}{1 - \cos\theta} = \frac{1}{\cos\theta} = \sec\theta$$ ✓

5. $\sec x - \cos x = \sin x \tan x$

$$LHS = \frac{1}{\cos x} - \cos x = \frac{1}{\cos x} - \cos x \cdot \frac{\cos x}{\cos x} = \frac{1}{\cos x} - \frac{\cos^2 x}{\cos x} = \frac{1 - \cos^2 x}{\cos x} = \frac{\sin^2 x}{\cos x}$$ ✓

$$RHS = \sin x \frac{\sin x}{\cos x} = \frac{\sin^2 x}{\cos x}$$ ✓

we rewrote both sides to meet in the middle

6. $\dfrac{1}{\sin x} - \dfrac{1}{\csc x} = \csc x - \sin x$

$LHS = \csc x - \sin x = RHS$ (too easy!)

7. $\dfrac{\tan x \cot x}{\cos x} = \sec x$

$$LHS = \frac{\frac{\sin x}{\cos x} \cdot \frac{\cos x}{\sin x}}{\cos x} = \frac{1}{\cos x} = \sec x$$