

## Formula Sheet for Pre-Calculus

### Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

### Co-function identities:

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$$

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$

$$\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$$

$$\sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta$$

$$\cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$$

### Even/odd Identities:

$$\sin(-\theta) = -\sin\theta$$

$$\cos(-\theta) = \cos\theta$$

$$\csc(-\theta) = -\csc\theta$$

$$\tan(-\theta) = -\tan\theta$$

$$\sec(-\theta) = \sec\theta$$

$$\cot(-\theta) = -\cot\theta$$

### Sum and difference formulas:

$$\sin(\theta \pm \varphi) = \sin\theta \cos\varphi \pm \cos\theta \sin\varphi$$

$$\cos(\theta \pm \varphi) = \cos\theta \cos\varphi \mp \sin\theta \sin\varphi$$

$$\tan(\theta \pm \varphi) = \frac{\tan\theta \pm \tan\varphi}{1 \mp \tan\theta \tan\varphi}$$

### Power reducing formulas:

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$\tan^2\theta = \frac{1-\cos 2\theta}{1+\cos 2\theta}$$

### Sum-to-product formulas:

$$\sin\theta + \sin\varphi = 2 \sin\left(\frac{\theta+\varphi}{2}\right) \cos\left(\frac{\theta-\varphi}{2}\right)$$

$$\sin\theta - \sin\varphi = 2 \cos\left(\frac{\theta+\varphi}{2}\right) \sin\left(\frac{\theta-\varphi}{2}\right)$$

$$\cos\theta + \cos\varphi = 2 \cos\left(\frac{\theta+\varphi}{2}\right) \cos\left(\frac{\theta-\varphi}{2}\right)$$

$$\cos\theta - \cos\varphi = -2 \sin\left(\frac{\theta+\varphi}{2}\right) \sin\left(\frac{\theta-\varphi}{2}\right)$$

### Product-to-sum formulas:

$$\sin\theta \sin\varphi = \frac{1}{2}[\cos(\theta - \varphi) - \cos(\theta + \varphi)]$$

$$\cos\theta \cos\varphi = \frac{1}{2}[\cos(\theta - \varphi) + \cos(\theta + \varphi)]$$

$$\sin\theta \cos\varphi = \frac{1}{2}[\sin(\theta + \varphi) + \sin(\theta - \varphi)]$$

$$\cos\theta \sin\varphi = \frac{1}{2}[\sin(\theta + \varphi) - \sin(\theta - \varphi)]$$

### Half-angle formulas:

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$$

### Double-angle formulas:

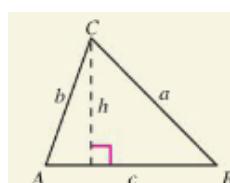
$$\sin(2\theta) = 2 \sin\theta \cos\theta$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

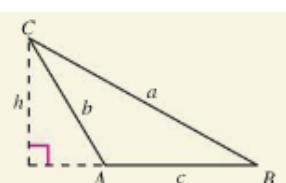
$$\tan(2\theta) = \frac{2 \tan\theta}{1 - \tan^2\theta}$$

### Law of Sines:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



A is acute.



A is obtuse.

### Law of Cosines:

$$a^2 = b^2 + c^2 - 2bc \cdot \cos(A)$$

$$b^2 = a^2 + c^2 - 2ac \cdot \cos(B)$$

$$c^2 = a^2 + b^2 - 2ab \cdot \cos(C)$$