

$$\sin t = y$$

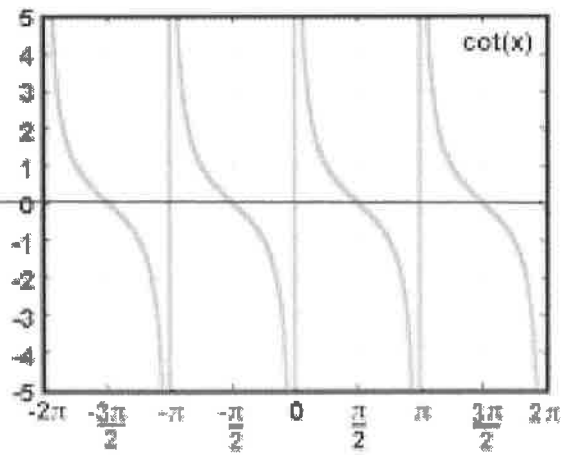
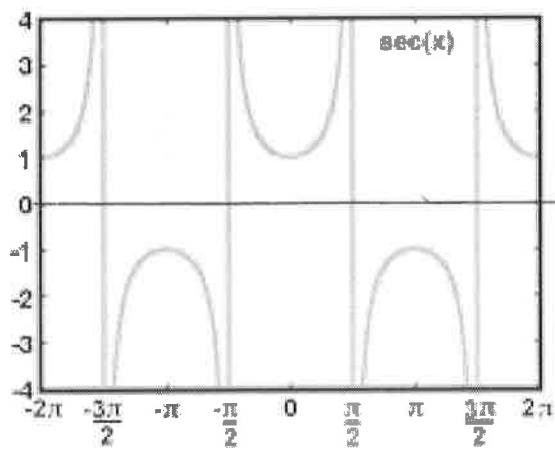
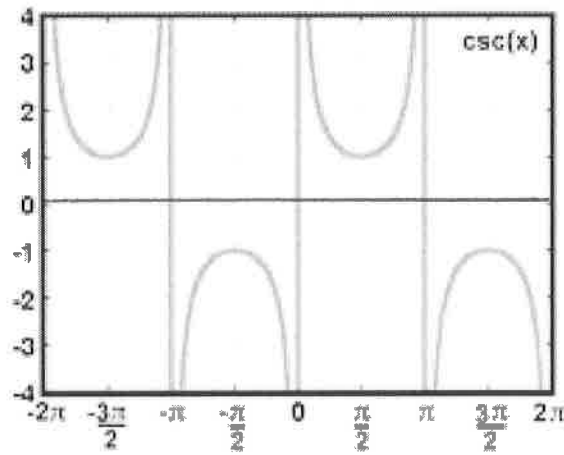
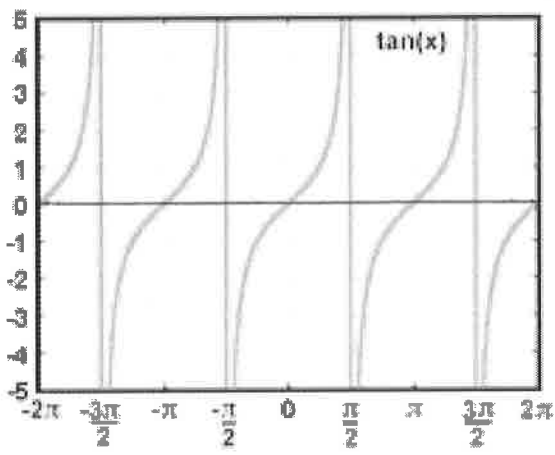
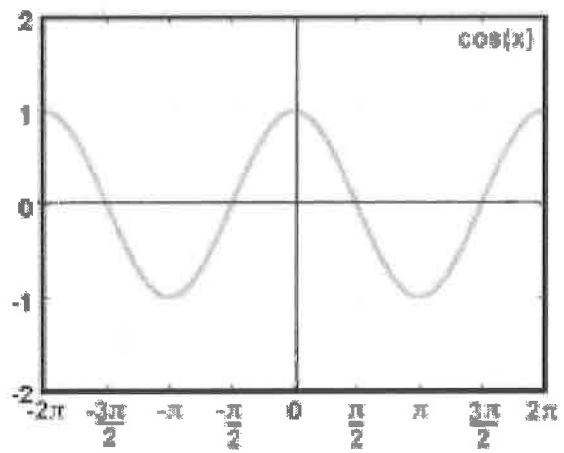
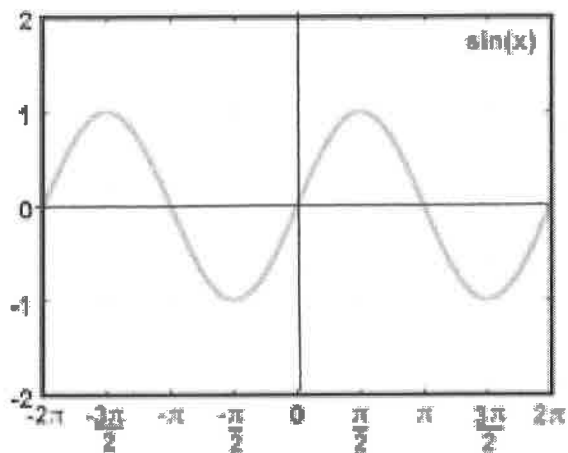
$$\cos t = x$$

$$\tan t = y/x, \text{ for } x \neq 0$$

$$\csc t = 1/y, y \neq 0$$

$$\sec t = 1/x, x \neq 0$$

$$\cot t = x/y, \text{ for } y \neq 0$$



	$y = a \sin(bx+d)+c$	$y = a \cos(bx+d)+c$	$y = a \csc(bx+d)+c$	$y = a \sec(bx+d)+c$	$y = a \tan(bx+d)+c$	$y = a \cot(bx+d)+c$
Period	$\frac{2\pi}{b}$			$\frac{\pi}{b}$		
Phase Shift	$bx+d=0$					
Key points						
Asymptotes						
Xmin Starting Point						
Xmax Ending Point						
Xscl						
Amplitude						
Vertical Shift						
Ymin						
Ymax						
Yscl						
Reflection $a < 0$ (Sketch)						

Trigonometric Formulas and Identities

The Fundamental Trigonometric Identities include:

Reciprocal Identities

$$\sin u = \frac{1}{\csc u} \quad \cos u = \frac{1}{\sec u} \quad \tan u = \frac{1}{\cot u}$$

$$\csc u = \frac{1}{\sin u} \quad \sec u = \frac{1}{\cos u} \quad \cot u = \frac{1}{\tan u}$$

Quotient Identities

$$\tan u = \frac{\sin u}{\cos u} \quad \cot u = \frac{\cos u}{\sin u}$$

Pythagorean Identities

$$\sin^2 u + \cos^2 u = 1 \quad 1 + \tan^2 u = \sec^2 u \quad 1 + \cot^2 u = \csc^2 u$$

Co-function Identities

$$\sin\left(\frac{\pi}{2} - u\right) = \cos u \quad \cos\left(\frac{\pi}{2} - u\right) = \sin u \quad \tan\left(\frac{\pi}{2} - u\right) = \cot u$$

$$\csc\left(\frac{\pi}{2} - u\right) = \sec u \quad \sec\left(\frac{\pi}{2} - u\right) = \csc u \quad \cot\left(\frac{\pi}{2} - u\right) = \tan u$$

Even/Odd Identities

$$\sin(-u) = -\sin u \quad \cos(-u) = \cos u \quad \tan(-u) = -\tan u$$

$$\csc(-u) = -\csc u \quad \sec(-u) = \sec u \quad \cot(-u) = -\cot u$$

Formulas include:

Sum and Difference Formulas

$$\sin(u + v) = \sin u \cos v + \cos u \sin v$$

$$\sin(u - v) = \sin u \cos v - \cos u \sin v$$

$$\cos(u + v) = \cos u \cos v - \sin u \sin v$$

$$\cos(u - v) = \cos u \cos v + \sin u \sin v$$

$$\tan(u + v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

$$\tan(u - v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

Double-Angle Formulas

$$\sin 2u = 2 \sin u \cos u$$

$$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u}$$

$$\cos 2u = \cos^2 u - \sin^2 u$$

$$= 2 \cos^2 u - 1$$

$$= 1 - 2 \sin^2 u$$

Power-Reducing Formulas

$$\sin^2 u = \frac{1 - \cos 2u}{2}$$

$$\cos^2 u = \frac{1 + \cos 2u}{2}$$

$$\tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$$

Half-Angle Formulas

$$\sin \frac{u}{2} = \pm \sqrt{\frac{1 - \cos u}{2}}$$

$$\cos \frac{u}{2} = \pm \sqrt{\frac{1 + \cos u}{2}}$$

$$\tan \frac{u}{2} = \frac{1 - \cos u}{\sin u} = \frac{\sin u}{1 + \cos u}$$

The signs of the sine and cosine depend on the quadrant that $u/2$ lies in.

Product-to-Sum Formulas

$$\sin u \sin v = \frac{1}{2} [\cos(u-v) - \cos(u+v)]$$

$$\cos u \cos v = \frac{1}{2} [\cos(u-v) + \cos(u+v)]$$

$$\sin u \cos v = \frac{1}{2} [\sin(u+v) + \sin(u-v)]$$

$$\cos u \sin v = \frac{1}{2} [\sin(u+v) - \sin(u-v)]$$

Sum-to-Product Formulas

$$\sin u + \sin v = 2 \sin\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right)$$

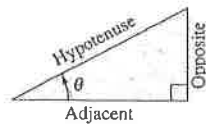
$$\sin u - \sin v = 2 \cos\left(\frac{u+v}{2}\right) \sin\left(\frac{u-v}{2}\right)$$

$$\cos u + \cos v = 2 \cos\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right)$$

$$\cos u - \cos v = -2 \sin\left(\frac{u+v}{2}\right) \sin\left(\frac{u-v}{2}\right)$$

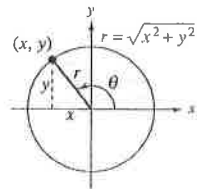
Definition of the Six Trigonometric Functions

Right triangle definitions, where $0 < \theta < \pi/2$



$$\begin{aligned} \sin \theta &= \frac{\text{opp.}}{\text{hyp.}} & \csc \theta &= \frac{\text{hyp.}}{\text{opp.}} \\ \cos \theta &= \frac{\text{adj.}}{\text{hyp.}} & \sec \theta &= \frac{\text{hyp.}}{\text{adj.}} \\ \tan \theta &= \frac{\text{opp.}}{\text{adj.}} & \cot \theta &= \frac{\text{adj.}}{\text{opp.}} \end{aligned}$$

Circular function definitions, where θ is any angle



$$\begin{aligned} \sin \theta &= \frac{y}{r} & \csc \theta &= \frac{r}{y} \\ \cos \theta &= \frac{x}{r} & \sec \theta &= \frac{r}{x} \\ \tan \theta &= \frac{y}{x} & \cot \theta &= \frac{x}{y} \end{aligned}$$

Reciprocal Identities

$$\begin{aligned} \sin u &= \frac{1}{\csc u} & \cos u &= \frac{1}{\sec u} & \tan u &= \frac{1}{\cot u} \\ \csc u &= \frac{1}{\sin u} & \sec u &= \frac{1}{\cos u} & \cot u &= \frac{1}{\tan u} \end{aligned}$$

Quotient Identities

$$\tan u = \frac{\sin u}{\cos u} \quad \cot u = \frac{\cos u}{\sin u}$$

Pythagorean Identities

$$\begin{aligned} \sin^2 u + \cos^2 u &= 1 \\ 1 + \tan^2 u &= \sec^2 u & 1 + \cot^2 u &= \csc^2 u \end{aligned}$$

Cofunction Identities

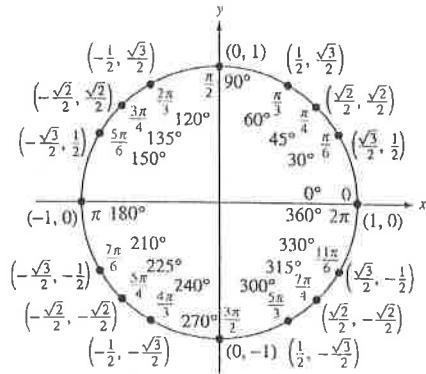
$$\begin{aligned} \sin\left(\frac{\pi}{2} - u\right) &= \cos u & \cot\left(\frac{\pi}{2} - u\right) &= \tan u \\ \cos\left(\frac{\pi}{2} - u\right) &= \sin u & \sec\left(\frac{\pi}{2} - u\right) &= \csc u \\ \tan\left(\frac{\pi}{2} - u\right) &= \cot u & \csc\left(\frac{\pi}{2} - u\right) &= \sec u \end{aligned}$$

Even/Odd Identities

$$\begin{aligned} \sin(-u) &= -\sin u & \cot(-u) &= -\cot u \\ \cos(-u) &= \cos u & \sec(-u) &= \sec u \\ \tan(-u) &= -\tan u & \csc(-u) &= -\csc u \end{aligned}$$

Sum and Difference Formulas

$$\begin{aligned} \sin(u \pm v) &= \sin u \cos v \pm \cos u \sin v \\ \cos(u \pm v) &= \cos u \cos v \mp \sin u \sin v \\ \tan(u \pm v) &= \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v} \end{aligned}$$



Double-Angle Formulas

$$\begin{aligned} \sin 2u &= 2 \sin u \cos u \\ \cos 2u &= \cos^2 u - \sin^2 u = 2 \cos^2 u - 1 = 1 - 2 \sin^2 u \\ \tan 2u &= \frac{2 \tan u}{1 - \tan^2 u} \end{aligned}$$

Power-Reducing Formulas

$$\begin{aligned} \sin^2 u &= \frac{1 - \cos 2u}{2} \\ \cos^2 u &= \frac{1 + \cos 2u}{2} \\ \tan^2 u &= \frac{1 - \cos 2u}{1 + \cos 2u} \end{aligned}$$

Sum-to-Product Formulas

$$\begin{aligned} \sin u + \sin v &= 2 \sin\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right) \\ \sin u - \sin v &= 2 \cos\left(\frac{u+v}{2}\right) \sin\left(\frac{u-v}{2}\right) \\ \cos u + \cos v &= 2 \cos\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right) \\ \cos u - \cos v &= -2 \sin\left(\frac{u+v}{2}\right) \sin\left(\frac{u-v}{2}\right) \end{aligned}$$

Product-to-Sum Formulas

$$\begin{aligned} \sin u \sin v &= \frac{1}{2} [\cos(u-v) - \cos(u+v)] \\ \cos u \cos v &= \frac{1}{2} [\cos(u-v) + \cos(u+v)] \\ \sin u \cos v &= \frac{1}{2} [\sin(u+v) + \sin(u-v)] \\ \cos u \sin v &= \frac{1}{2} [\sin(u+v) - \sin(u-v)] \end{aligned}$$