Chapter Eleven: Techniques of Differentiation with Applications

11.1 Derivatives of Powers, Sums, and Constant Multiples

Now we can use shortcuts. If I ask you to compute the derivative using the definition, you must use the limit definition. Otherwise, you may use these rules.

Power Rule If *n* is any constant and $f(x)=x^n$, then $f'(x)=nx^{n-1}$.

Differential Notation - Differentiation $\frac{d}{dx}$ means "the derivative with respect to x." Thus, $\frac{d}{dx}\left[f(x)\right]$ is the same thing as *f′(x)*, the derivative of *f(x)* with respect to *x*. If *y* is a function of *x*, then the derivative of *y* with respect to *x* is $\frac{d}{dx}(y)$ or, more compactly, $\frac{dy}{dx}$.

Derivatives of Sums, Differences, and Constant Multiples If *f(x)* and *g(x)* are any two differentiable functions, and if *c* is any constant, then the funtions *f(x)±g(x)* and *cf(x)* are differentiable and

$(f(x)\pm g(x))'=f'(x)\pm g'(x)$	Sum Rule
$(cf(x))'=cf'(x)$	Constant Multiple Rule

In words:

The derivative of a sum is the sum of the derivatives, and the derivative of a difference is the difference of the derivatives.

The derivative of *c* times a function is *c* times the derivative of the function.

Differential Notation:

$$\frac{d}{dx}(f(x)\pm g(x))=\frac{d}{dx}(f(x))\pm\frac{d}{dx}(g(x))$$

$$\frac{d}{dx}\left[cf(x)\right]=c\frac{d}{dx}(f(x))$$

So we can now find the derivative of all polynomials.

$*$ **Constant times *x* and a constant** If *c* is any constant, then $\frac{d}{dx}(cx)=c$ and $\frac{d}{dx}(c)=0$. $*$ Remember these!

Derivative of |x|: $\frac{d}{dx}|x|=\frac{|x|}{x}$. Note that the derivative does not exist when *x=0*.

Examples: Find the derivatives.

1. $f(x)=x^4$

   $f'(x)=4x^{4-1}=4x^3$

2. $f(x)=x^{-1}$

   $f'(x)=-1x^{-1-1}=-1x^{-2}=-x^{-2}$

3. $f(x)=-x^3-3x^2-1$

   $f'(x)=-(3x^{3-1})-3(2x^{2-1})-0$ derivative of the constant 1 is 0.

   $f'(x)=-3x^2-6x$

4. $f(x)=\frac{2}{x}-\frac{2}{x^3}+\frac{1}{x^4}$

   Rewrite: $f(x)=2x^{-1}-2x^{-3}+1x^{-4}$

   $f'(x)=2(-1x^{-1-1})-2(-3x^{-3-1})+1(-4x^{-4-1})$

   $f'(x)=-2x^{-2}+6x^{-4}-4x^{-5}$

   Rewrite: $f'(x)=\frac{-2}{x^2}+\frac{6}{x^4}-\frac{4}{x^5}$

5. $t(x)=3|x|-\sqrt{x}$

   Rewrite: $t(x)=3|x|-x^{1/2}$

   $t'(x)=3\frac{|x|}{x}-\frac{1}{2}x^{1/2-1}$

   $t'(x)=\frac{3|x|}{x}-\frac{1}{2}x^{-1/2}$

   Rewrite: $t'(x)=\frac{3|x|}{x}-\frac{1}{2x^{1/2}}=\frac{3|x|}{x}-\frac{1}{2\sqrt{x}}$

Examples: Find the equation of the tangent line to the graph of the given function at the given point.

slope = derivative

$y - y_1 = m(x - x_1)$

1. $f(x) = x^4; (-2, 16)$

$f'(x) = 4x^{4-1} = 4x^3$

$m = f'(-2) = 4(-2)^3 = 4(-8) = -32$

$y - 16 = -32(x - (-2))$

$y - 16 = -32(x + 2)$

$y - 16 = -32x - 64$

$y = -32x - 48$

2. $f(x) = \frac{1}{x^2}; x = 1$

evaluate: $f(1) = \frac{1}{(1)^2} = 1$ to get point $(1,1)$

Rewrite: $f(x) = x^{-2}$

$f'(x) = -2x^{-2-1} = -2x^{-3} = \frac{-2}{x^3}$

$m = f'(1) = \frac{-2}{(1)^3} = -\frac{2}{1} = -2$

$y - 1 = -2(x - 1)$

$y - 1 = -2x + 2$

$y = -2x + 3$

Examples: Find all values of *x* (if any) where the tangent line to the graph is horizontal.

$f'(x) = 0$

1. $y = -3x^2 - x$

derivative of $-x$ is -1

$y' = -3(2x) - 1$

$y' = -6x - 1$

$0 = -6x - 1$

$1 = -6x$

$\frac{1}{-6} = x$

2. $y = x - \sqrt{x}$ $= x - x^{1/2}$

$y' = 1 - \frac{1}{2}x^{1/2 - 1} = 1 - \frac{1}{2}x^{-1/2} = 1 - \frac{1}{2x^{1/2}} = 1 - \frac{1}{2\sqrt{x}}$

$0 = 1 - \frac{1}{2\sqrt{x}}$

$2\sqrt{x} \cdot \frac{1}{2\sqrt{x}} = 1 \cdot 2\sqrt{x}$

$1 = 2\sqrt{x}$

$\frac{1}{2} = \sqrt{x}$

$\left(\frac{1}{2}\right)^2 = (\sqrt{x})^2$

$\frac{1}{4} = x$

L'Hospital's Rule If *f* and *g* are two differentiable functions such that substituting *x=a* in the expression $\frac{f(x)}{g(x)}$ gives either $\frac{0}{0}$ or $\frac{\infty}{\infty}$, then $\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}$. That is, we can replace *f*(*x*) and *g*(*x*) with their derivatives and try again to take the limit.

Examples: Use L'Hospitals' Rule, if necessary.

1. $\lim_{x\to -1}\frac{x^2+3x+2}{x^2+x} = \frac{(-1)^2+3(-1)+2}{(-1)^2+(-1)} = \frac{1-3+2}{1-1} = \frac{0}{0}$ use L'Hospital's rule

$\lim_{x\to -1}\frac{2x+3}{2x+1} = \frac{2(-1)+3}{2(-1)+1} = \frac{-2+3}{-2+1} = \frac{1}{-1} = -1$

2. $\lim_{x\to 2}\frac{x^2-3x+2}{x^2-x-2} = \frac{(2)^2-3(2)+2}{(2)^2-(2)-2} = \frac{4-6+2}{4-2-2} = \frac{0}{0}$ use the rule

$\lim_{x\to 2}\frac{2x-3}{2x-1} = \frac{2(2)-3}{2(2)-1} = \frac{4-3}{4-1} = \frac{1}{2}$

3. $\lim_{x\to 1}\frac{x^2-4x+5}{x^3-1} = \frac{(1)^2-4(1)+5}{(1)^3-1} = \frac{1-4+5}{1-1} = \frac{2}{0}$

Cannot use the rule!

to evaluate this limit we could use a table:

X	.99	.999	1	1.001	1.01
f(x)	-68.0	-668.0	?	665.3	65.3

$\xrightarrow{\quad}$ − Big $\qquad$ $\xleftarrow{\quad}$ + Big

the limit does not exist (dne)