11.3 The Product and Quotient Rules

Product Rule If *f*(*x*) and *g*(*x*) are differentiable functions of *x*, then so is their product *f(x)g(x)*, and $\frac{d}{dx}\left[f(x)g(x)\right]=f'(x)g(x)+f(x)g'(x)$. In words, the derivative of the product is the derivative of the first times the second, plus the first times the derivative of the second.

Examples: Find the derivative.

1. $y=3x^2(2x+1)$

$f(x)=3x^2$ $\quad g(x)=2x+1$

$f'(x)=6x$ $\quad g'(x)=2$

$y'=6x(2x+1)+3x^2(2)$

$y'=12x^2+6x+6x^2$ or $y'=18x^2+6x$

2. $y=\left(x^{0.7}-4x-5\right)\left(x^{-1}+x^{-2}\right)$

$f(x)=x^{0.7}-4x-5$ $\quad g(x)=x^{-1}+x^{-2}$

$f'(x)=0.7x^{-0.3}-4$ $\quad g'(x)=-x^{-2}-2x^{-3}$

Parentheses are important

$y'=(0.7x^{-0.3}-4)(x^{-1}+x^{-2})+(x^{0.7}-4x-5)(-x^{-2}-2x^{-3})$

no way am I simplifying this one!!

3. $f(x)=\sqrt{x}\left(x^5+3x^2-x^{-3}\right)$

$f(x)=\sqrt{x}=x^{1/2}$ $\quad g(x)=x^5+3x^2-x^{-3}$

$f'(x)=\frac{1}{2}x^{-1/2}$ $\quad g'(x)=5x^4+6x+3x^{-4}$

$f'(x)=\frac{1}{2}x^{-1/2}(x^5+3x^2-x^{-3})+\sqrt{x}(5x^4+6x+3x^{-4})$

4. $f(x)=\left(x^2-3x+|x|\right)\left(\frac{1}{x^3}-\frac{7}{x^2}+3\right)$

$f(x)=x^2-3x+|x|$ $\quad g(x)=\frac{1}{x^3}-\frac{7}{x^2}+3=x^{-3}-7x^{-2}+3$

$f'(x)=2x-3+\frac{|x|}{x}$ $\quad g'(x)=-3x^{-4}+14x^{-3}$

$f'(x)=\left(2x-3+\frac{|x|}{x}\right)\left(\frac{1}{x^3}-\frac{7}{x^2}+3\right)+\left(x^2-3x+|x|\right)\left(-3x^{-4}+14x^{-3}\right)$

Quotient Rule If *f*(*x*) and *g*(*x*) are differentiable functions of *x*, then so is their quotient *f(x)/g*(*x*) (provided *g(x)≠0*), and $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)=\frac{g(x)f'(x)-f(x)g'(x)}{\left[g(x)\right]^2}$. In words, the derivative of the quotient is the bottom derivative of the top, minus the top times the derivative of the bottom, all over the bottom squared. Or, lodehi - hidelo over lo squared.

Examples: Find the derivative.

1. $y=\frac{3x-9}{2x+4}$

lo = $2x+4$ hi = $3x-9$

delo = 2 dehi = 3

$$y'=\frac{(2x+4)(3)-(3x-9)(2)}{(2x+4)^2}$$

Note: the extensive use of parentheses are important!

2. $y=\frac{3x^2-9x+11}{4x-1}$

lo = $4x-1$ hi = $3x^2-9x+11$

delo = 4 dehi = $6x-9$

$$y'=\frac{(4x-1)(6x-9)-(3x^2-9x+11)(4)}{(4x-1)^2}$$

Another note: the $(4x-1)$ from the first part does not cancel with the denominator because it is not found after the subtraction

3. $f(x)=\frac{\sqrt{x}+7}{2x^3-14x^2+x}$

lo = $2x^3-14x^2+x$ hi = $\sqrt{x}+7$

delo = $6x^2-28x+1$ dehi = $\frac{1}{2}x^{-1/2}$

$$f'(x)=\frac{(2x^3-14x^2+x)(\frac{1}{2}x^{-1/2})-(\sqrt{x}+7)(6x^2-28x+1)}{(2x^3-14x^2+x)^2}$$

4. $f(x)=\frac{x^{-6}+7x^3-1}{|x|}$

lo = $|x|$ hi = $x^{-6}+7x^3-1$

delo = $\frac{|x|}{x}$ dehi = $-6x^{-7}+21x^2$

$$f'(x)=\frac{|x|(-6x^{-7}+21x^2)-(x^{-6}+7x^3-1)(\frac{|x|}{x})}{(|x|)^2}$$