

11.5 Derivatives of Logarithmic and Exponential Functions

Derivative of the Natural Logarithm $\frac{d}{dx}[\ln x] = \frac{1}{x}$

$$\frac{d}{dx} \ln u = \frac{u'}{u} = \frac{1}{u} \frac{du}{dx}$$

Derivatives of all Logs $\frac{d}{dx}[\log_b x] = \frac{1}{x \ln b}$

$$\frac{d}{dx} \log_b u = \frac{u'}{u \ln b} = \frac{1}{u \ln b} \frac{du}{dx}$$

Chain rule for logarithms:
derivative of inside divided by inside

Examples: Find the derivative.

1. $f(x) = \ln(x+3)$

$$f'(x) = \frac{1}{x+3}$$

inside = $x+3$
derivative of inside = $1+0=1$

2. $f(x) = \log_3 x$

$$f'(x) = \frac{1}{x \ln 3}$$

3. $f(x) = \ln(3x^2 - 7x)$

$u = 3x^2 - 7x$
der of $u = 6x - 7$

$$f'(x) = \frac{6x-7}{3x^2-7x}$$

this is simplified!

4. $g(x) = \log_5(17x-12)$

$u = 17x - 12$
der = 17

$$g'(x) = \frac{17}{(17x-12) \ln 5}$$

$$5. g(x) = \ln\left(\frac{x}{2x-1}\right)$$

$$u = \frac{x}{2x-1}$$

$$\text{der} = \frac{(2x-1)(1) - x(2)}{(2x-1)^2} = \frac{2x-1-2x}{(2x-1)^2} = \frac{-1}{(2x-1)^2}$$

$$g'(x) = \frac{\frac{-1}{(2x-1)^2}}{\frac{x}{2x-1}} = \frac{-1}{(2x-1)^2} \cdot \frac{2x-1}{x} = \frac{-1}{x(2x-1)}$$

divide by a fraction is multiply by reciprocal

$$6. g(x) = \ln|2x-4|$$

$$u = |2x-4|$$

$$\text{der} = \frac{|2x-4|}{2x-4} \cdot 2 = \frac{2|2x-4|}{2x-4} = \frac{|2x-4|}{x-2}$$

$$\frac{\cancel{2}|2x-4|}{\cancel{2}(x-2)}$$

$$g'(x) = \frac{\frac{|2x-4|}{x-2}}{|2x-4|} = \frac{|2x-4|}{x-2} \cdot \frac{1}{|2x-4|} = \frac{1}{x-2}$$

Derivative of e^x : $\frac{d}{dx} e^x = e^x$

$$\frac{d}{dx} e^u = e^u \frac{du}{dx} = u' e^u$$

Chain rule:
derivative of exponent times original

Derivative of b^x : $\frac{d}{dx} [b^x] = b^x \ln b$

$$\frac{d}{dx} b^u = b^u \ln b \frac{du}{dx} = u' b^u \ln b$$

notice the $\ln b$ again...

Examples: Find the derivative.

$$1. h(x) = e^{x^2}$$

exponent = x^2
derivative = $2x$

$$h'(x) = 2x e^{x^2}$$

$$2. f(x) = e^{1-x}$$

$$\begin{aligned} \text{exponent} &= 1-x \\ \text{derivative} &= -1 \end{aligned}$$

$$f'(x) = -1 e^{1-x}$$

$$3. g(x) = e^{3x(4x-7)}$$

$$\begin{aligned} \text{exponent} &= 3x(4x-7) \\ \text{derivative} &= 3(4x-7) + 3x(4) \\ &= 12x - 21 + 12x \\ &= 24x - 21 \end{aligned}$$

$$g'(x) = (24x - 21) e^{3x(4x-7)}$$

↑
parenthesis so entire u' is multiplied by e^u .

$$4. g(x) = 3^{x^2-x}$$

$$\begin{aligned} \text{exp} &= x^2 - x \\ \text{derivative} &= 2x - 1 \end{aligned}$$

$$g'(x) = (2x-1) 3^{x^2-x} \ln 3$$

$$5. f(x) = 7^{|2x+5|}$$

$$\begin{aligned} \text{exp} &= |2x+5| \\ \text{derivative} &= \frac{|2x+5|}{2x+5} \cdot 2 = \frac{2|2x+5|}{2x+5} \end{aligned}$$

nothing cancels!

$$f'(x) = \frac{2|2x+5|}{2x+5} 7^{|2x+5|} \cdot \ln 7$$