12.4 Analyzing Graphs

Features of a Graph:

1. The x- and y-intercepts: If $y = f(x)$, find the x-intercepts by setting $y = 0$ and solving for x; find the y-intercept by setting $x = 0$ and solving for y.

2. Relative Extrema: Use the technique of section 12.1 to find relative extrema.

3. Points of Inflection: Use the techniques of section 12.3 to find points of inflection.

4. Behavior near points where the function is not defined: If $f(x)$ is not defined at $x = a$, consider $\lim_{x\to a^-} f(x)$ and $\lim_{x\to a^+} f(x)$ to see how the graph behaves as x approaches a.

5. Behavior at infinity: Consider $\lim_{x\to\pm\infty} f(x)$, if appropriate, to see how the graph behaves to the left and the right as x gets extremely large.

Examples: Sketch the graph, label everything.

1. $f(x) = -x^2 - 2x - 1$

Intercepts use original function

y-int is $f(0) = -1$

x-int is $0 = -x^2 - 2x - 1$

too hard, skip it.

Critical $x = -1$

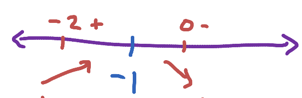
$f(-1) = 0$

$f'(x) = -2x - 2$

$0 = -2x - 2$

$2x = -2$

$x = -1$ critical point



$f'(-2) = \text{pos}$ $\quad f'(0) = \text{neg}$

increasing $(-\infty, -1)$

decreasing $(-1, \infty)$

points

$(0,-1)$

$(-1,0)$

$f''(x) = -2$

no points of inflection

f'' is always negative

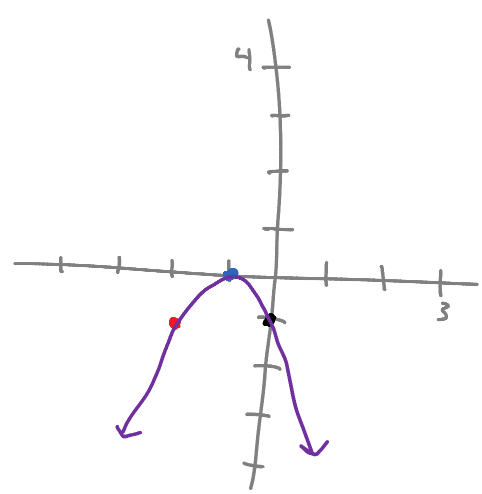
so always concave down

not a rational function

so no worries about limits

extra point as needed: $x = -2$

$f(-2) = -(-2)^2 - 2(-2) - 1 = -1$



2. $f(x) = 4x^3 + 3x^2 + 2$

$(0, 2)$ y-int is $f(0) = 2$

x-int too hard

$f'(x) = 12x^2 + 6x$

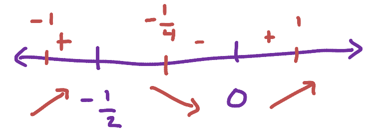
$0 = 6x(2x+1)$

$6x = 0$ or $2x + 1 = 0$

$x = 0$ or $x = -\frac{1}{2}$

$f(0) = 2$ $(0, 2)$ min

$f(-\frac{1}{2}) = 2.25$ $(-.5, 2.25)$ max



$f'(-1) = 12 - 6 = pos$

$f'(-\frac{1}{4}) = \frac{12}{16} - \frac{3}{2} = neg$

$f'(1) = 12 + 6 = pos$

inc $(-\infty, -\frac{1}{2}), (0, \infty)$

dec $(-\frac{1}{2}, 0)$

$f''(x) = 24x + 6$

$0 = 24x + 6$

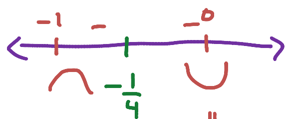
$-6 = 24x$

$-\frac{6}{24} = x = -\frac{1}{4}$

$f(-\frac{1}{4}) = 2.125$ $(-0.25, 2.125)$ point of inflection

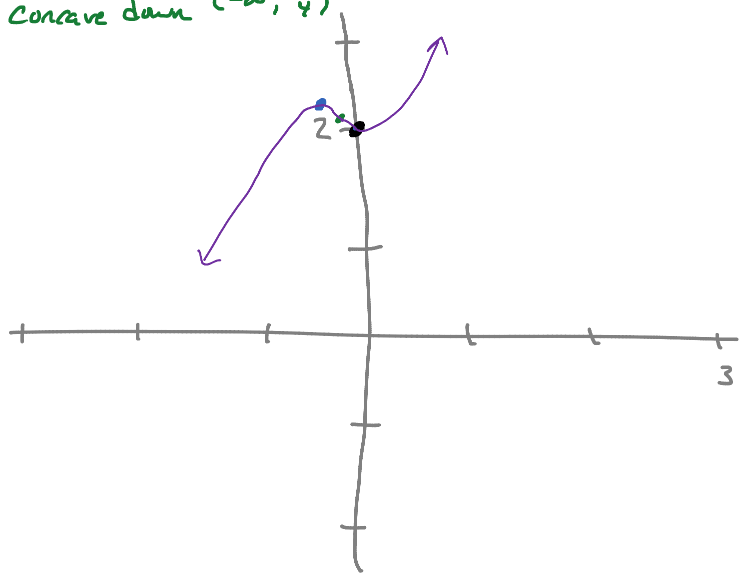
concave up $(-\frac{1}{4}, \infty)$

concave down $(-\infty, -\frac{1}{4})$



$f''(-1) = neg$ $f''(0) = pos$

not rational so no need to check limits



3. $f(x) = x^2 + \frac{1}{x^2}$ ← $x=0$ not in domain of f.

no y-intercept

no x-intercept

$f(x) = x^2 + x^{-2}$

critical pts

$(-1, 2)$ $f(-1) = 2$

$(1, 2)$ $f(1) = 2$

$f'(x) = 2x - 2x^{-3}$

$f'(x) = 2x - \frac{2}{x^3}$

undefined at $x=0$

$0 = 2x - \frac{2}{x^3}$

$\frac{2}{x^3} = 2x \to 2 = 2x^4 \to 1 = x^4 \to x = \pm 1$

not a point

min min

$-2 \quad -\frac{1}{2} \quad \frac{1}{2} \quad 2$

$-1 \quad 0 \quad 1$

$f'(-2) = -4 + \frac{2}{8} = \text{neg}$ $f'(\frac{1}{2}) - 1 - 16 - \text{neg}$

$f'(-\frac{1}{2}) = -1 + 16 - \text{pos}$ $f'(2) = 4 - \frac{2}{8} - \text{pos}$

Increasing $(-1, 0), (1, \infty)$

decreasing $(-\infty, -1), (0, 1)$

$f''(x) = 2 + 6x^{-4}$

$f''(x) = 2 + \frac{6}{x^4}$

undefined at $x=0$

$0 = 2 + \frac{6}{x^4}$

$-2 = \frac{6}{x^4}$ never

so $f''(x) \neq 0$

$f'(x) = 2 + \frac{6}{x^4}$ is always positive so

Concave up $(-\infty, 0), (0, \infty)$

Let's look more at $x=0$:

$\lim_{x \to 0^-} (x^2 + \frac{1}{x^2}) = 0 + \infty = \text{Big positive}$

$\lim_{x \to 0^-} (x^2 + \frac{1}{x^2}) = 0 + \infty = \text{Big positive}$

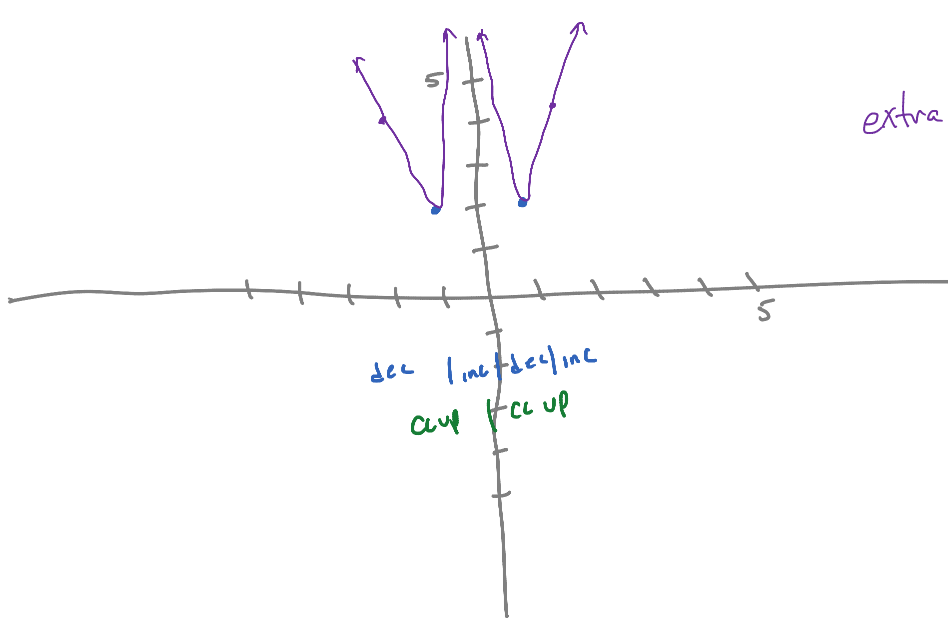
vertical asymptote at $x=0$

Let's look at $\pm\infty$:

$\lim_{x \to -\infty} (x^2 + \frac{1}{x^2}) = \infty + 0 = \text{Big positive}$

$\lim_{x \to \infty} (x^2 + \frac{1}{x^2}) = \infty + 0 = \text{big positive}$

no horizontal asymptote, both ends finish going up.



extra points:

$x = -2, y = 4.25$

$x = 2, y = 4.25$