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Abstract: *The subject of this paper is the numerical simulation of the interaction of two–dimensional incompressible viscous flow and a vibrating airfoil. A solid elastically supported airfoil with two degrees of freedom, which can rotate around the elastic axis and oscillate in the vertical direction, is considered. The numerical simulation consists of the stabilized finite element solution of the Reynolds averaged Navier–Stokes equations with algebraic models of turbulence, coupled with the system of ordinary differential equations describing the airfoil motion. Since the computational domain is time dependent and the grid is moving, the Arbitrary Lagrangian-Eulerian (ALE) method is used. The developed method was applied to the simulation of flow induced airfoil vibrations.*

AMS subject classification: XXXXX, YYYYY, ZZZZZ

Keywords: aeroelasticity, stabilized finite element method, ALE formulation, algebraic turbulent models, flow induced airfoil vibrations, flutter

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1 Introduction

The interaction of fluid flow and elastic structures plays an important role in many technical disciplines - airplane industry (e.g., wings vibrations), blade machines (oscillations of blades in turbines and compressors), civil engineering (interaction of a strong wind with TV towers, cooling towers or bridges), etc. The research in aeroelasticity or hydroelasticity focuses on the interaction between flowing fluids and vibrating structures (see, e.g. [5] and [10]).

In [15], the problem of the flow induced airfoil vibrations is analyzed with the aid of the finite element method. The finite element solution of the Navier–Stokes equations written in the ALE form is coupled with the numerical solution of the system of ordinary differential equations describing the airfoil motion.

Since the Reynolds number is rather high (10^5–10^7), it is necessary to take into account the effects caused by turbulence. The present paper represents an extesion of the methods and results obtained in [15] by including a model of turbulence in the simulation process. We apply and test here the algebraic models of turbulence designed by Baldwin and Lomax ([1]) and by Rostand ([12]). As a result we obtain a sufficiently accurate and robust method, which we apply to the simulation of flow induced airfoil vibrations.

2 Continuous problem

We assume that $(0, T)$ is a time interval and by $\Omega_t \subset I\!R^2$ we denote a bounded computational domain occupied by the fluid at time t. The symbols $\boldsymbol{u} = \boldsymbol{u}(x, t)$ and $p = p(x, t),\ x \in \Omega_t,\ t \in (0, T)$, denote the flow velocity and the kinematic pressure (i.e., dynamic pressure divided by the density ρ of the fluid) and ν denotes the kinematic viscosity. We have $\boldsymbol{u} = (u_1, u_2)$, where u_1 and u_2 are the components of the velocity in the directions of the Cartesian coordinates x_1 and x_2 of x. By $I\!R$ and $I\!R^2$ we denote the set of all real numbers and the set of all two-dimensional vectors, respectively.

The character of the flow depends on the magnitude of the Reynolds number $Re = Uc/\nu$, where U denotes the characteristic velocity (in our case the magnitude of the far field velocity) and c is the characteristic length (here the length of the chord of the airfoil). The flow with a sufficiently small Reynolds number is laminar. If the Reynolds number is larger than some critical value, the flow becomes turbulent.

We use the models of turbulence based on the so-called Reynolds averaged equations and the Boussinesq hypothesis ([13]), which yield the Reynolds averaged equations in the form

$$\nabla \cdot \boldsymbol{u} = 0, \tag{2.1}$$
$$\frac{\partial u_i}{\partial t} + (\boldsymbol{u} \cdot \nabla)\, u_i + \frac{\partial p}{\partial x_i} - \sum_{j=1}^{2} \frac{\partial}{\partial x_j} \Big\{ (\nu + \nu_T) \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \Big\} = 0, \quad i = 1, 2.$$

Here ν_T is the so-called turbulent viscosity. It is computed on the basis of various turbulence models.

System (2.1) is equipped with the initial condition

$$\boldsymbol{u}(x, 0) = \boldsymbol{u}_0, \quad x \in \Omega_0, \tag{2.2}$$

and boundary conditions

$$\text{a)} \qquad \boldsymbol{u}|_{\Gamma_D} = \boldsymbol{u}_D, \qquad \text{b)} \quad \boldsymbol{u}|_{\Gamma_{Wt}} = \boldsymbol{w}|_{\Gamma_{Wt}}, \tag{2.3}$$
$$\text{c)} \qquad -(p - p_{ref})\, n_i + (\nu + \nu_T) \sum_{j=1}^{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) n_j = 0 \quad \text{on } \Gamma_O, \quad i = 1, 2.$$

Here $\boldsymbol{n} = (n_1, n_2)$ is the unit outer normal to the boundary $\partial\Omega_t$ of the domain Ω_t, Γ_D represents the inlet (and, possibly, fixed impermeable walls), Γ_O is the outlet and Γ_{Wt} is the boundary of the airfoil at time t. Condition (2.3), b) represents the assumption that the fluid adheres to the airfoil moving with the velocity $\boldsymbol{w}|_{\Gamma_{Wt}}$. By p_{ref} we denote a prescribed reference (far field) pressure.

The vertical displacement H (oriented downwards) and rotation α (oriented clockwise) of the airfoil are described by the system ([15])

$$\begin{aligned} m\ddot{H} + S_\alpha \ddot{\alpha} \cos\alpha + k_{HH} H + d_{HH} \dot{H} - S_\alpha \dot{\alpha}^2 \sin\alpha &= -L(t), \\ S_\alpha \ddot{H} \cos\alpha + I_\alpha \ddot{\alpha} + k_{\alpha\alpha} \alpha + d_{\alpha\alpha} \dot{\alpha} &= M(t), \end{aligned} \tag{2.4}$$

where m denotes the mass of the airfoil, S_α, I_α are the static moment and the inertia moment around the elastic axis, k_{HH}, $k_{\alpha\alpha}$ denote the bending stiffness and the torsional stiffness, d_{HH}, $d_{\alpha\alpha}$ are the structural dampings. The aerodynamic lift force $L(t)$ and the aerodynamic torsional moment $M(t)$ are defined by the relations

$$L = -\ell \int_{\Gamma_{Wt}} \sum_{j=1}^{2} \tau_{2j} n_j dS, \quad M = \ell \int_{\Gamma_{Wt}} \sum_{i,j=1}^{2} \tau_{ij} n_j r_i^{\text{ort}} dS, \tag{2.5}$$
$$\tau_{ij} = \rho \left[-p\delta_{ij} + \nu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right], \quad r_1^{\text{ort}} = -(x_2 - x_{EO2}), \; r_2^{\text{ort}} = x_1 - x_{EO1}.$$

The symbol ℓ denotes the depth of the airfoil. These relations determine the interaction between the moving fluid and the airfoil.

3 Discrete problem

3.1 ALE method

In order to simulate flow in a moving domain, we employ the Arbitrary Lagrangian-Eulerian (ALE) method (cf. [11]), based on a one-to-one ALE mapping

$$\mathcal{A}_t : \Omega_{\mathrm{ref}} \to \Omega_t, \quad X \mapsto x(X,t) = \mathcal{A}_t(X), \tag{3.1}$$

of the reference configuration $\Omega_{\mathrm{ref}} = \Omega_0$ onto the current configuration Ω_t, with the domain velocity $\boldsymbol{w} = \partial \mathcal{A}_t / \partial t$. We suppose that the domain velocity at each point on the surface of the airfoil is equal to the velocity of its motion. By $\frac{D^{\mathcal{A}}}{Dt}$ we denote the ALE derivative - i.e. derivative with respect to the reference configuration. This means that for a function $f : \{(x,t); x \in \Omega_t, t \in [0,T]\} \to I\!R$ and $x = \mathcal{A}_t(X), X \in \Omega_{\mathrm{ref}}$, we introduce the function $\tilde{f}(X,t) = f(\mathcal{A}_t(X),t)$ and define

$$\frac{D^{\mathcal{A}} f}{Dt}(x,t) = \frac{\partial \tilde{f}}{\partial t}(X,t). \tag{3.2}$$

With the aid of the ALE method we rewrite system (2.1) in the form

$$\nabla \cdot \boldsymbol{u} = 0, \tag{3.3}$$

$$\frac{D^{\mathcal{A}}}{Dt} u_i + ((\boldsymbol{u} - \boldsymbol{w}) \cdot \nabla)\, u_i + \frac{\partial p}{\partial x_i} - \sum_{j=1}^{2} \frac{\partial}{\partial x_j} \Big\{ (\nu + \nu_T) \, \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \Big\} = 0, \quad i = 1,2.$$

3.2 Time discretization

We consider a partition $0 = t_0 < t_1 < \cdots < T$, $t_k = k\tau$ of the time interval $[0,T]$ with a time step $\tau > 0$ and use the approximations $\boldsymbol{u}(t_n) \approx \boldsymbol{u}^n$ and $p(t_n) \approx p^n$ of the exact solution and $\boldsymbol{w}(t_n) \approx \boldsymbol{w}^n$ of the domain velocity at time t_n. Using the second-order backward difference formula for the approximation of the ALE derivative and setting $\hat{\boldsymbol{u}}^i = \boldsymbol{u}^i \circ \mathcal{A}_{t_i} \circ \mathcal{A}^{-1}_{t_{n+1}}$, we obtain the system

$$\nabla \cdot \boldsymbol{u}^{n+1} = 0, \tag{3.4}$$

$$\frac{3u_i^{n+1} - 4\hat{u}_i^n + \hat{u}_i^{n-1}}{2\tau} + ((\boldsymbol{u}^{n+1} - \boldsymbol{w}^{n+1}) \cdot \nabla)\, u_i^{n+1} + \frac{\partial p^{n+1}}{\partial x_i} \tag{3.5}$$

$$- \sum_{j=1}^{2} \frac{\partial}{\partial x_j} \Big\{ (\nu + \nu_T(\boldsymbol{u}^{n+1})) \left(\frac{\partial u_i^{n+1}}{\partial x_j} + \frac{\partial u_j^{n+1}}{\partial x_i} \right) \Big\} = 0, \quad i = 1,2, \quad \text{in } \Omega_{t_{n+1}}.$$

3.3 Finite element space discretization

The starting point for the space discretization by the finite element method is the weak formulation. For simplicity we set $\Omega = \Omega_{t_{n+1}}, \boldsymbol{u} = \boldsymbol{u}^{n+1}, p = p^{n+1}$. We define the function spaces $W = (H^1(\Omega))^2$, $X = \{\boldsymbol{v} \in W; \boldsymbol{v}|_{\Gamma_D \cup \Gamma_{W_t}} = 0\}$, $Q = L^2(\Omega)$, where $L^2(\Omega)$ is the Lebesgue space of square integrable functions over the domain Ω and $H^1(\Omega) = \{v \in L^2(\Omega); \frac{\partial v}{\partial x_i} \in L^2(\Omega),\, i = 1, 2\}$ is the Sobolev space. By $(\cdot, \cdot)_\Omega$ we denote the $L^2(\Omega)$-scalar product.

The weak formulation is obtained in a standard way. Equation (3.4) is multiplied by a test function $q \in Q$ and equation (3.5) is multiplied by a test function $\boldsymbol{v} \in X$, integrated over the domain Ω, Green's theorem is applied, the boundary condition (2.3), c) is used and the resulting integral identities are summed. Under the notation $\boldsymbol{U} = (\boldsymbol{u}, p), \boldsymbol{U}^* = (\boldsymbol{u}^*, p), \boldsymbol{V} = (\boldsymbol{v}, q)$ and

$$\begin{aligned} a(\boldsymbol{U}^*, \boldsymbol{U}, \boldsymbol{V}) = {} & \frac{3}{2\tau}(\boldsymbol{u}, \boldsymbol{v})_\Omega + \left(\left((\boldsymbol{u}^* - \boldsymbol{w}^{n+1}) \cdot \nabla\right) \boldsymbol{u}, \boldsymbol{v}\right)_\Omega - (p, \operatorname{div} \boldsymbol{v})_\Omega \\ & + \sum_{i,j=1}^{2} \int_\Omega \frac{1}{2}(\nu + \nu_T) \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) \mathrm{d}x + (\operatorname{div} \boldsymbol{u}, q)_\Omega, \end{aligned} \tag{3.6}$$

$$f(\boldsymbol{V}) = \frac{1}{2\tau}\left(4\hat{\boldsymbol{u}}^n - \hat{\boldsymbol{u}}^{n-1}, \boldsymbol{v}\right)_\Omega - \int_{\Gamma_O} p_{ref}\, \boldsymbol{n} \cdot \boldsymbol{v} \,\mathrm{d}S, \tag{3.7}$$

we define a weak solution as a couple $\boldsymbol{U} = (\boldsymbol{u}, p) \in W \times Q$ such that $\boldsymbol{u}$ satifies the boundary conditions (2.3), a)-b), and

$$a(\boldsymbol{U}, \boldsymbol{U}, \boldsymbol{V}) = f(\boldsymbol{V}) \quad \forall\, \boldsymbol{V} = (\boldsymbol{v}, q) \in X \times Q. \tag{3.8}$$

On the basis of the weak formulation we introduce a finite element solution. We approximate the spaces W, X, Q by subspaces W_h, X_h, Q_h, $h \in (0, h_0), h_0 > 0$, where $X_h = W_h \cap X$. The approximate solution is defined as a couple $\boldsymbol{U}_h = (\boldsymbol{u}_h, p_h) \in W_h \times Q_h$ such that

$$a(\boldsymbol{U}_h, \boldsymbol{U}_h, \boldsymbol{V}_h) = f(\boldsymbol{V}_h), \quad \forall\, \boldsymbol{V}_h = (\boldsymbol{v}_h, q_h) \in X_h \times Q_h \tag{3.9}$$

and $\boldsymbol{u}_h$ satisfies an approximation of the boundary conditions (2.3), a)-b).

The finite element spaces are constructed in the following way. Let Ω be a polygonal domain and $\mathcal{T}_h$ its triangulation formed by a finite number of closed triangles with standard properties from the finite element method. We use the well-known Taylor-Hood P_2/P_1 elements:

$$p \approx p_h \in Q_h = \{q \in Q \cap C(\bar{\Omega});\, q|_K \in P_1(K), \forall K \in \mathcal{T}_h\} \tag{3.10}$$

and

$$\boldsymbol{u} \approx \boldsymbol{u}_h \in W_h = \{\boldsymbol{v} \in W \cap (C(\bar{\Omega}))^2;\, \boldsymbol{v}|_K \in (P_2(K))^2, \forall K \in \mathcal{T}_h\}, \quad X_h = W_h \cap X.$$

Here $P_k(K)$ denotes the space of all polynomials on $K \in \mathcal{T}_h$ of degree $\leq k$. The spaces $X_h,\ Q_h$ satisfy the Babuska-Brezzi condition (cf. [7]).

3.4 Stabilized finite element method

For large Reynolds numbers our problem becomes singularly perturbed with dominating convection and approximate solutions can contain nonphysical spurious oscillations. In order to avoid them, we apply the stabilization by the streamline–diffusion method combined with the pressure stabilization. We set $\overline{\boldsymbol{w}} = \boldsymbol{u}^* - \boldsymbol{w}^{n+1}$ and for $\boldsymbol{U} = (\boldsymbol{u}, p)$, $\boldsymbol{U}^* = (\boldsymbol{u}^*, p)$, $\boldsymbol{V} = (\boldsymbol{v}, q)$ introduce the stabilization forms

$$\mathcal{L}_h(\boldsymbol{U}^*, \boldsymbol{U}, \boldsymbol{V}) = \tag{3.11}$$

$$= \sum_{K \in \mathcal{T}_h} \delta_K \left(\frac{3}{2\tau} \boldsymbol{u} - \sum_{j=1}^{2} \frac{\partial}{\partial x_j} \Big\{ (\nu + \nu_T) \, (\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}) \Big\} + (\overline{\boldsymbol{w}} \cdot \nabla) \, \boldsymbol{u} + \nabla p, \, (\overline{\boldsymbol{w}} \cdot \nabla) \, \boldsymbol{v} \right)_K , \tag{3.12}$$

$$\mathcal{F}_h(\boldsymbol{V}) = \sum_{K \in \mathcal{T}_h} \delta_K \left(\frac{1}{2\tau} (4 \hat{\boldsymbol{u}}^n - \hat{\boldsymbol{u}}^{n-1}), \, (\overline{\boldsymbol{w}} \cdot \nabla) \, \boldsymbol{v} \right)_K , \tag{3.13}$$

and

$$\mathcal{P}_h(\boldsymbol{U}, \boldsymbol{V}) = \sum_{K \in \mathcal{T}_h} \tau_K \, (\nabla \cdot \boldsymbol{u}, \nabla \cdot \boldsymbol{v})_K \, . \tag{3.14}$$

The symbol $(\cdot, \cdot)_K$ denotes the scalar product in the space $L^2(K)$ and $\delta_K \geq 0$ and $\tau_K \geq 0$ are suitable parameters.

The stabilized discrete problem is formulated in the following way. Find $\boldsymbol{U}_h = (\boldsymbol{u}_h, p_h) \in W_h \times Q_h$ such that $\boldsymbol{u}_h$ satisfies an approximation of the boundary conditions (2.3), a)-b) and

$$a(\boldsymbol{U}_h, \boldsymbol{U}_h, \boldsymbol{V}_h) + \mathcal{L}_h(\boldsymbol{U}_h, \boldsymbol{U}_h, \boldsymbol{V}_h) + \mathcal{P}_h(\boldsymbol{U}_h, \boldsymbol{V}_h) = f(\boldsymbol{V}_h) + \mathcal{F}_h(\boldsymbol{V}_h)$$
$$\text{for all } \boldsymbol{V}_h = (\boldsymbol{v}_h, q_h) \in X_h \times Q_h. \tag{3.15}$$

Parameters δ_K and τ_K are chosen according to [9] and [6].

The discrete stabilized nonlinear problem (3.15) at time $t = t_{n+1}$ is solved with the aid of the Oseen iterations

$$a(\boldsymbol{U}_h^{(\ell)}, \boldsymbol{U}_h^{(\ell+1)}, \boldsymbol{V}_h) + \mathcal{L}_h(\boldsymbol{U}_h^{(\ell)}, \boldsymbol{U}_h^{(\ell+1)}, \boldsymbol{V}_h) + \mathcal{P}_h(\boldsymbol{U}_h^{(\ell+1)}, \boldsymbol{V}_h) = f(\boldsymbol{V}_h) + \mathcal{F}_h(\boldsymbol{V}_h) \tag{3.16}$$
$$\text{for all } \boldsymbol{V}_h = (\boldsymbol{v}_h, q_h) \in X_h \times Q_h, \quad \ell = 0, 1, \ldots \, .$$

The initial approximation $\boldsymbol{U}_h^{(0)}$ is defined on the basis of the approximate solution on the previous time level t_n. Problem (3.16) is equivalent to a system of linear algebraic equations, which is solved by the direct solver UMFPACK ([3]).

The computation of the force L and the moment M at time $t = t_{n+1}$ from the approximate solution is carried out with aid of a weak reformulation similarly as in [14].

4 Algebraic models of turbulence

Now it remains to specify the evaluation of the turbulent viscosity ν_T. For this purpose we use algebraic models of turbulence.

4.1 Model Cebeci–Smith (CS model)

A basis for the application of algebraic models of turbulence is the Cebeci–Smith model ([2]). In this model, the computational domain Ω is divided in an inner domain near the wall (airfoil), where the internal turbulent viscosity ν_{Ti} is evaluated. In the outer part of the computational domain the outer turbulence viscosity ν_{To} is computed. In practical computations, both viscosities are evaluated in the whole domain Ω and then the turbulent viscosity is defined as

$$\nu_T = \min(\nu_{Ti}, \nu_{To}). \tag{4.1}$$

The turbulent viscosity is computed with the aid of a local coordinate system (X, Y), where X is measured along the airfoil or along the axis of the wake and the axis Y is orthogonal to X. Components of the velocity in the directions X and Y are denoted by U and V. The inner turbulent viscosity

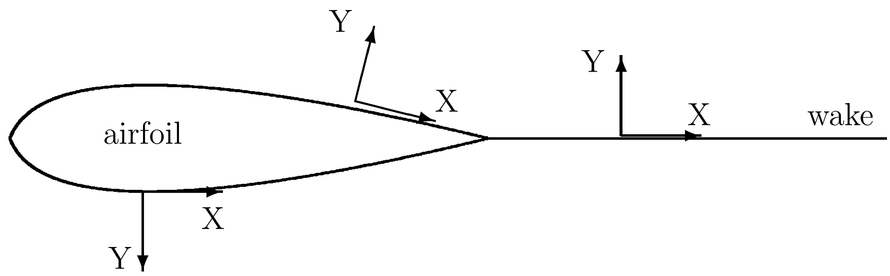

Figure 1: Local coordinates X, Y

is defined by

$$\nu_{Ti} = \rho\, l^2 \left|\frac{\partial U}{\partial Y}\right|, \tag{4.2}$$

where ρ is the fluid density and l is the mixing length, which is determined by the relations

$$l = \kappa\, Y\, F_D, \quad F_D = 1 - \exp\left(-\frac{1}{A^+}\,\frac{u_\tau\, Y}{\nu}\right). \tag{4.3}$$

F_D is the so-called van Driest function. The symbol $Y = Y(x)$ denotes the orthogonal distance of a point $x \in \Omega$ from the airfoil or the axis of the wake. The so-called shear velocity u_τ is defined by

$$u_\tau = \left(\nu \left|\frac{\partial U}{\partial Y}\right|\right)_w^{\frac{1}{2}}. \tag{4.4}$$

The subscript w means the value for $Y = 0$. The empirical constant $A^+ = 26$ was determined for $\kappa = 0.4$ from experimental data obtained from the flow around a plate. In the wake we set $\nu_{Ti} = \infty$.

The outer turbulent viscosity is defined by the Clauser relation

$$\nu_{To} = \rho\, \alpha\, \delta_i^*\, U_e\, F_k, \tag{4.5}$$

where $\alpha = 0.0168$,

$$F_k = \left[1 + 5.5\left(\frac{Y}{\delta}\right)^6\right]^{-1}, \tag{4.6}$$

δ is the shear layer thickness, $U_e = U(\delta)$ represents the velocity of the outer flow and δ_i^* is the kinematic displacement thickness

$$\delta_i^* = \int_0^\delta \left(1 - \frac{U}{U_e}\right) \mathrm{d}Y. \tag{4.7}$$

The given algebraic model requires to know the quantities δ, $U_e\delta_i^*$ and u_τ, which is not suitable for the solution of our problem. Therefore, in the sequel we consider some modifications of the CS model.

4.2 Baldwin–Lomax model (BL model)

Baldwin and Lomax ([1]) proposed the following modification of the CS model. The inner turbulent viscosity is defined as

$$\nu_{Ti} = \rho\, l^2\, |\omega|, \tag{4.8}$$

where

$$\omega = \frac{\partial U}{\partial Y} - \frac{\partial V}{\partial X} \tag{4.9}$$

is the vorticity. The outer turbulent viscosity is determined by the relations

$$\nu_{To} = \rho\, \alpha\, C_{cp}\, F_w\, F_k, \tag{4.10}$$

$$F_w = \min\left(F_{w_1}, F_{w_2}\right), \quad F_{w_1} = Y_{\max} F_{\max}, \quad F_{w_2} = C_{wk}\, Y_{\max} \frac{(\triangle U)^2}{F_{\max}}. \tag{4.11}$$

The value $F_{\max}$ is the maximum of the function $F = Y\,|\omega|\,F_D$ on the line $X = \text{const.}$, $Y \geq 0$, $Y_{\max}$ is the value, for which $F(Y_{\max}) = F_{\max}$ and $\triangle U$ is the maximum variation of the velocity U along the line $X = \text{const.}$, $Y \geq 0$. For the Klebanoff function F_k we have

$$F_k = \left[1 + 5.5\left(C_{KL}\frac{Y}{Y_{\max}}\right)^6\right]^{-1}. \tag{4.12}$$

Further, the shear layer thickness δ is expessed as $\delta = Y_{\max}/C_{KL}$. Baldwin and Lomax use the following constants $\alpha = 0.0168$, $C_{cp} = 1.6$, $C_{KL} = 0.3$, $C_{wk} = 0.25$.

We also applied the Rostand model ([12]), which gives comparable results.

5 Flow induced airfoil vibrations

This section presents results of the numerical simulation of flow induced vibrations obtained for the airfoil NACA 0012. The following quantities, used in [15], are considered: $m = 0.086622$ kg, $S_\alpha = -0.000779673$ kg m, $I_\alpha = 0.000487291$ kg m^2, $k_{HH} = 105.109$ N/m, $k_{\alpha\alpha} = 3.695582$ N m/rad, $\ell = 0.05$ m, $c = 0.3$ m, $\rho = 1.225$ kg/m^3, $\nu = 1.5 \cdot 10^{-5}$ m/s^2. The positions of the elastic axis and the centre of gravity of the airfoil measured along the chord from the leading edge are $x_{EO1} = 0.4c = 0.12$ m, and $x_{T1} = 0.37c = 0.111$ m, respectively. The coefficients of the proportional damping are considered in the form $d_{HH} = \varepsilon\, k_{HH}$ and $d_{\alpha\alpha} = \varepsilon\, k_{\alpha\alpha}$, where we choose $\varepsilon = 10^{-3}$.

The computational process for the solution of the nonstationary problem is based on the coupling of the fluid flow problem in the discrete form (3.15) with the numerical solution of the nonlinear structural model (2.5) by the 4th order Runge-Kutta method. It starts at time $t = 0$ by the solution of the flow, keeping the airfoil in a fixed position given by the prescribed initial translation $H_0 = 10$ mm and the angle of attack $\alpha_0 = -7°$. Then, at time $\delta t > 0$ the airfoil is released and we continue by the solution of a complete fluid-structure interaction problem with $H(\delta t) = H_0$, $\dot{H}(\delta t) = 0$, $\alpha(\delta t) = \alpha_0$, $\dot{\alpha}(\delta t) = 0$. The computational mesh was constructed in an adaptive way with the aid of the package ANGENER ([4]).

The simulation of the airfoil motion due to the fluid-structure interaction in time domain is shown in Figures 2 – 4 for the far field velocity $U_\infty = 30$, 35 and 40 m/s.

For far field flow velocity $U_\infty \leq 35$ m/s) the displacement H and the rotation angle α are dying in time. When U_∞ exeeds 30 m/s, then the damping of flow induced vibrations becomes weaker and at about $U_\infty \approx 40$ m/s the system becomes unstable by flutter, the vibrations amplitudes are becoming very high and exeed more than 100 mm and 15° at a limit cycle oscillation (see Figure 4). Such behaviour of the system is in a general agreement with the previous numerical simulations carried out in [15], where no turbulence was considered, but a different airfoil type was modeled.

6 Conclusion

In this paper the numerical method for the simulation of the interaction of viscous incompressible turbulent flow and a vibrating airfoil is developed. The method contains several important ingredients:

- ALE method for the treatment of the time dependent computational domain,

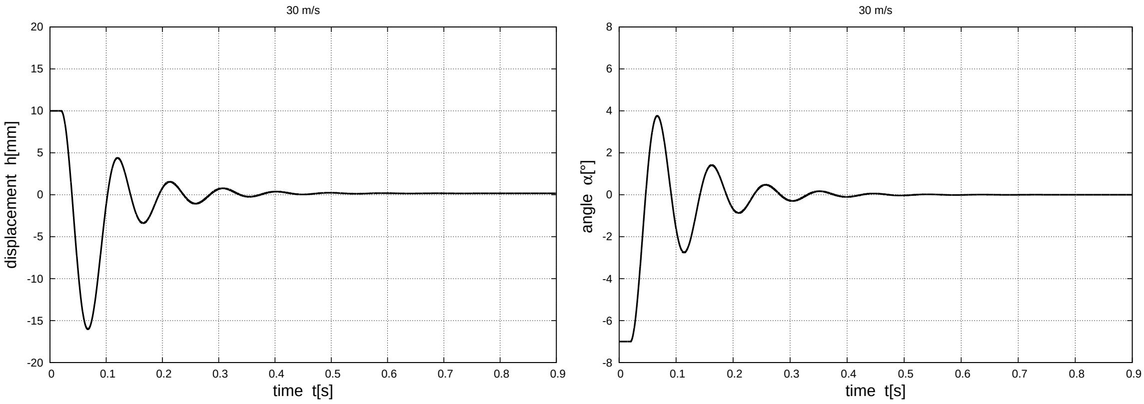

Figure 2: Flow induced airfoil vibrations for $U_\infty = 30$ m/s

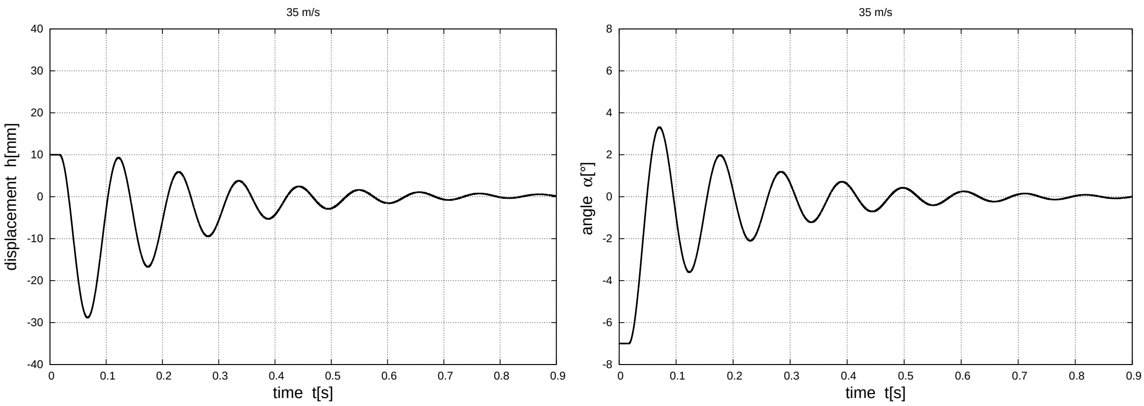

Figure 3: Flow induced airfoil vibrations for $U_\infty = 35$ m/s

- time discretization using second-order backward difference formula,
- space discretization by the stabilized finite elements, satisfying the Babuska–Brezzi condition,
- the use of the Reynolds averaged system of equations and the application of algebraic models of turbulence.

The method was applied to the solution of the flow induced vibrations of the profile NACA 0012. The results prove that the developed method allows the simulation of the interaction of turbulent flow with large Reynolds numbers and the airfoil vibrations with large deformations.

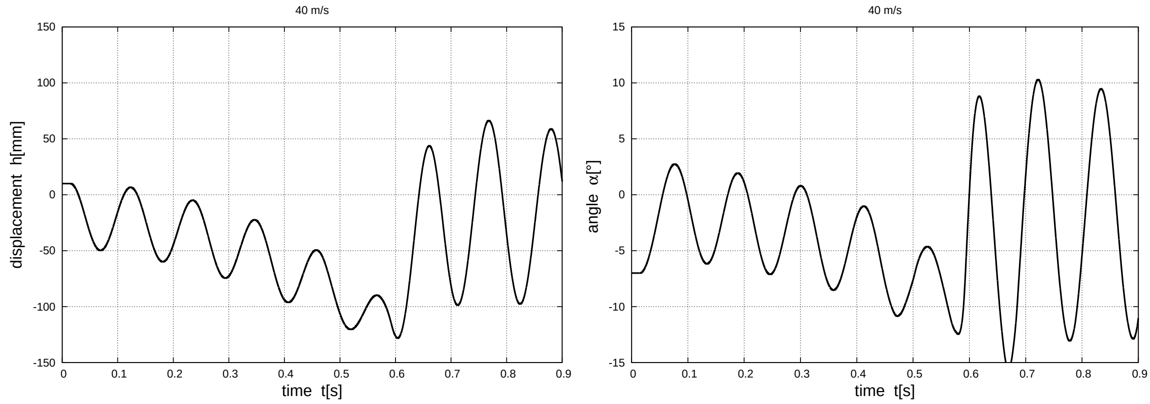

Figure 4: Flow induced airfoil vibrations for $U_\infty = 40$ m/s

The next step in the research will be a further investigation of turbulence models included in the described technique. From the point of view of practical applications it will be suitable to increase the speed of the computational process by the use of the domain decomposition method and parallelization of the algorithm. A more detailed comparison of the turbulent and laminar flow in the relation to the system behaviour is needed.

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